Sub-Kelvin Feedback Cooling and Heating Dynamics of an Optically Levitated Librator

Fons van der Laan,[1,*] Felix Tebbenjohanns,[1] René Reimann,[1,2] Jayadev Vijayan,[1] Lukas Novotny,[1] and Martin Frimmer[1]
[1]*Photonics Laboratory, ETH Zürich, 8093 Zürich, Switzerland*
[2]*Quantum Research Centre, Technology Innovation Institute, Abu Dhabi, UAE*

(Received 25 December 2020; revised 15 April 2021; accepted 17 August 2021; published 16 September 2021)

Rotational optomechanics strives to gain quantum control over mechanical rotors by harnessing the interaction of light and matter. We optically trap a dielectric nanodumbbell in a linearly polarized laser field, where the dumbbell represents a nanomechanical librator. Using measurement-based parametric feedback control in high vacuum, we cool this librator from room temperature to 240 mK and investigate its heating dynamics when released from feedback. We exclude collisions with residual gas molecules as well as classical laser noise as sources of heating. Our findings indicate that we observe the torque fluctuations arising from the zero-point fluctuations of the electromagnetic field.

DOI: 10.1103/PhysRevLett.127.123605

Introduction.—Harnessing the quantum properties of light to control mechanical motion is a central theme of optomechanics [1–3]. The paradigmatic optomechanical system is a light field interacting with a mechanical degree of freedom coupled to a thermal bath. In the quantum regime, the coupling between the mechanics and the light is sufficiently strong to overcome the interaction with the thermal bath. In this regime, the quantum properties of light can be exploited to perform measurements at, and even below, the standard quantum limit [4,5]. Furthermore, this regime allows for measurement-based feedback control of the mechanics outside the bounds of classical physics [6,7], a prerequisite for engineering massive objects into macroscopic quantum states [8,9].

For translational motion, the hallmark signature of the quantum nature of light dominating the dynamics of the mechanics has been the observation of radiation pressure shot noise [10,11]. These force fluctuations arise due to the quantization of linear momentum of light into units of $\hbar k$ (with k the wave number) [12]. Alternatively, this effect can be viewed as an interference of the deterministic measurement field with the vacuum fluctuations [13].

With linear mechanical motion under quantum control, rotational motion has attracted increasing attention in optomechanics in recent years [14,15]. While technology platforms have been developed that transduce radiation pressure fluctuations into angular motion, these systems are insensitive to the angular momentum carried by the light [16,17]. In contrast, optically levitated nanoparticles are ideal test beds for rotational optomechanics [18–21]. In a linearly polarized field, an anisotropic particle aligns to the polarization direction, making this system an optically levitated librator, i.e., a rotor with a linear restoring force [22,23]. The application of techniques developed to cool translational motion [24,25] offers promise to turn libration modes into quantum resources. A particularly exciting prospect is the engineering of quantum revivals to explore quantum mechanics at a macroscopic scale [26–28].

The ideal starting point for rotational quantum optomechanics is the librational ground state. To reach that point, two challenges must be met. First, techniques must be developed to cool librational motion more efficiently than the current state of the art [21]. Second, nanomechanical librators must be transitioned to a regime where the quantum noise of the electromagnetic field dominates their angular heating. This heating can be understood in analogy to heating of linear motion. Besides linear momentum, light also carries angular momentum [29]. While the quantization of light's linear momentum gives rise to radiation pressure shot noise, the field's spin quantization must give rise to torque fluctuations (termed radiation torque shot noise) in the interaction with a mechanical object [30–32]. This torque noise can be understood as the particle's dipole moment (induced by the driving field) interacting with the field's vacuum fluctuations in the orthogonal polarization direction, as illustrated in Fig. 1(a). Alternatively, radiation torque shot noise can be understood as the inevitable measurement backaction arising from the fact that the orientation of an anisotropic particle can be inferred from the scattered field. Entering this backaction-dominated regime is thus a necessary requirement to exert quantum control over an optically levitated librator.

In this work, we feedback cool the librational motion of an optically trapped nanodumbbell to a temperature of 240 mK using parametric feedback control. Furthermore, we investigate the source of the heating dynamics of the libration after cooling is switched off. In high vacuum, our experiments rule out thermal fluctuations from the gas and technical laser noise as the dominant heating mechanisms. The measured heating rate agrees within a factor of 3 with that predicted by theory for radiation torque shot noise.

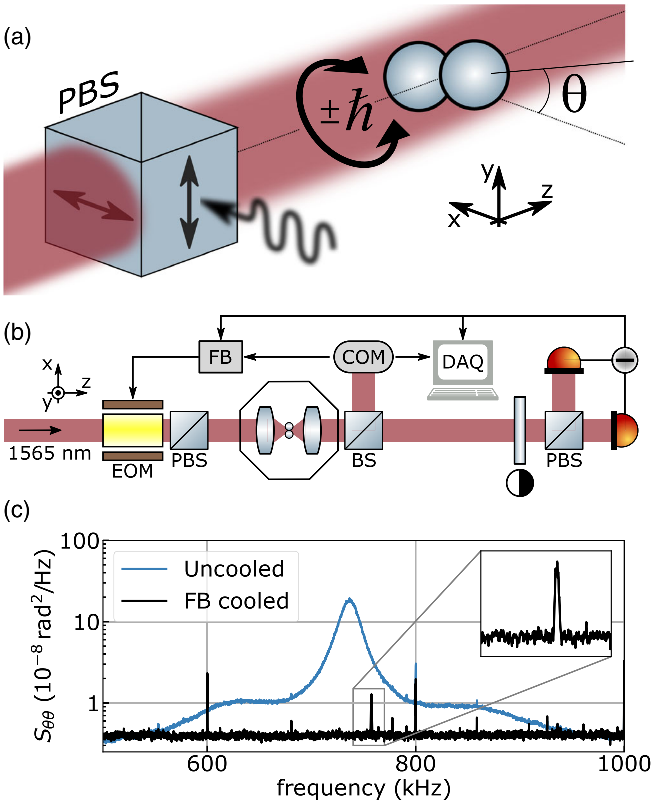

FIG. 1. (a) Pictorial representation of radiation torque shot noise. An anisotropic scatterer in a linearly polarized light field experiences a fluctuating torque that arises from the vacuum fluctuations, illustrated as entering the unused port of the polarizing beam splitter (PBS). (b) Schematic of the experimental setup. Inside a vacuum chamber, we focus a laser beam (propagating along z, linearly polarized along x) with an aspheric lens to form an optical trap. In the forward direction, the light is collected and split into two paths with a beam splitter (BS). One half of the optical power is sent to a center-of-mass (COM) motion detector. The other half is used to measure the libration angle θ in a balanced detection scheme. The measurement is recorded with a data-acquisition card (DAQ). The intensity of the laser beam (focal power $P = 1205(23)$ mW) is modulated with an electro-optic modulator (EOM) using feedback signals derived from the COM and the libration detector, respectively. (c) The blue line shows the measured power spectral density $S_{\theta\theta}$ of the uncooled libration motion at a pressure of $p_{\mathrm{gas}} = 7.0(7)$ mbar. The broad spectrum is a result of coupling between the angular degrees of freedom. The black line shows $S_{\theta\theta}$ at $p_{\mathrm{gas}} = 1.1(1) \times 10^{-8}$ mbar and with feedback cooling engaged for COM and librational motion, where the signal of the libration detector reduces to a single resonant line. The additional peaks are electronic noise from our data-acquisition card.

Experimental system.—Our experimental setup is shown in Fig. 1(b). We trap a dumbbell (composed of two silica spheres, nominal diameter 136 nm; for details on the characterization, see the Supplemental Material [33]) in a strongly focused laser beam (wavelength $\lambda = 1565.0(1)$ nm). The beam propagates along the z axis and is linearly polarized along the x axis. The laser power in the optical trap can be controlled with an electro-optical modulator. In the forward direction, the light from the trap is collected with a lens and divided at a beam splitter. Half of the signal is sent to a center-of-mass (COM) motion detector [44]. The other half is sent through a polarizing beam splitter and onto a balanced photodiode to detect the angular orientation of the dumbbell [19,20,45]. For small deviation angles of the dumbbell relative to the polarization axis, our detection scheme is sensitive to the angle θ of the dumbbell relative to the x axis in the focal xy plane [46]. Furthermore, the restoring torque generated by the light field on the dumbbell is to first order linear in θ. The dumbbell is thus a harmonic oscillator with a libration frequency Ω_l, following the equation of motion

$$I\ddot{\theta} + I\gamma\dot{\theta} + I\Omega_l^2\theta = \tau_{\mathrm{fl}}, \tag{1}$$

with I the moment of inertia of the dumbbell, γ the damping rate, and each dot indicating a time derivative. The fluctuating torque τ_{fl} drives the librator. In this Letter, we provide evidence that at low pressures τ_{fl} is dominated by the vacuum fluctuations of the light field.

The measured power spectral density $S_{\theta\theta}$ of the orientation angle θ at a pressure $p_{\mathrm{gas}} = 7.0(7)$ mbar at room temperature is shown in Fig. 1(c) in blue. The spectrum resembles a resonant line shape, centered at 750 kHz, flanked by two broad shoulders on either side. This spectral shape has been explained as a consequence of the intricate rotational dynamics of the dumbbell, where the thermally driven spinning degree of freedom around the long axis of the dumbbell gives rise to an interaction between the two other orientational degrees of freedom [23,46]. We detail the calibration procedure for our detector signal in the Supplemental Material [33].

At pressures $p_{\mathrm{gas}} < 10^{-4}$ mbar, the gas damping is sufficiently low to apply effective feedback cooling to the libration and the center-of-mass motion. For both types of motion, we use the parametric feedback-cooling scheme originally developed for COM cooling [11] and suggested for libration cooling [46]. In this cooling technique, a phase-locked loop tracks the detector signal to generate a feedback signal at twice the oscillation frequency of the measured degree of freedom. This feedback signal is applied to the modulator controlling the power of the trapping laser to generate a periodic modulation of the optical potential. A spectrum $S_{\theta\theta}$ under feedback cooling at $p_{\mathrm{gas}} = 1.1(1) \times 10^{-8}$ mbar is shown in Fig. 1(c) in black. Under feedback cooling, the spectrum of the libration reduces to a single line centered at $\Omega_l = 2\pi \times 757$ kHz. The observed linewidth is limited by drifts of Ω_l arising from slow drifts of the laser power. The area under the peak is a measure for the energy of the librator, and we extract a value of $E = 0.24(3)$ K. Note that throughout this Letter, we normalize all energies by the Boltzmann constant to have the unit Kelvin. This energy is a result of the balance of damping γ and heating by the fluctuating torque τ_{fl} acting on the librator.

Reheating protocol.—To quantify the torque fluctuations driving the levitated librator, we perform reheating experiments [47]. Each measurement cycle starts with the librator under feedback cooling. At time $t = 0$, we turn off the feedback for the libration (while center-of-mass cooling remains engaged) and measure the energy in the libration mode (extracted from the spectrum $S_{\theta\theta}$) as a function of time. The cycle repeats as we reengage feedback cooling of the libration. Since each experimental run records one realization of the stochastic reheating process, we repeat the cycle 400 times. In Fig. 2(a), we show $S_{\theta\theta}$ averaged over all cycles at $p_{\mathrm{gas}} = 1.1(1) \times 10^{-8}$ mbar at the beginning ($t = 0$ ms) of the reheating period, and in (b) at the end ($t = 950$ ms). We extract the energy of the librator (indicated by the blue shaded area) by integrating the power spectrum after subtracting the noise floor (gray area). The resulting mean libration energy is shown as a function of time in Fig. 2(c). The heating process for the mean energy E follows the equation

$$E(t) = E_0 + (E_\infty - E_0)(1 - \mathrm{e}^{-\gamma t}), \qquad (2)$$

with $E_0 = E(t = 0)$ and E_∞ the energy the system is equilibrating to. On a short timescale $t \ll \gamma^{-1}$, and for $E_\infty \gg E_0$, we find $E(t) = \gamma E_\infty t$. Thus, the slope of a linear fit to the data in Fig. 2(c) yields the heating rate $\Gamma = \gamma E_\infty$.

Results and discussion.—Having established our protocol to measure the heating rate Γ of the levitated librator, we now investigate the origin of the fluctuating torque driving the reheating. To quantify the contribution from the interaction with the residual gas in the vacuum chamber, we plot the measured heating rate Γ as a function of gas pressure in Fig. 3 as blue data points. At pressures above 10^{-7} mbar, the heating rate scales linearly with pressure, as indicated by the dotted line. This scaling is expected, since the fluctuating torque due to the gas scales linearly with pressure. At pressures below 10^{-7} mbar, we observe a significant deviation of the measured heating rate from the linear scaling, and Γ approaches a constant value. We fit our data with the function $\Gamma = a \times p_{\mathrm{gas}} + \Gamma_{\mathrm{res}}$, shown as the solid black curve in Fig. 3, with the proportionality constant a and the residual heating rate Γ_{res} as fit parameters. We obtain $\Gamma_{\mathrm{res}} = 0.51$ K/s. At a pressure $p_{\mathrm{gas}} = 1.1(1) \times 10^{-8}$ mbar, the residual torque noise acting on the dumbbell exceeds the thermal torque noise by a factor of 4. The observation of the heating rate being independent of pressure supports the hypothesis that Γ_{res} is dominated by radiation torque shot noise.

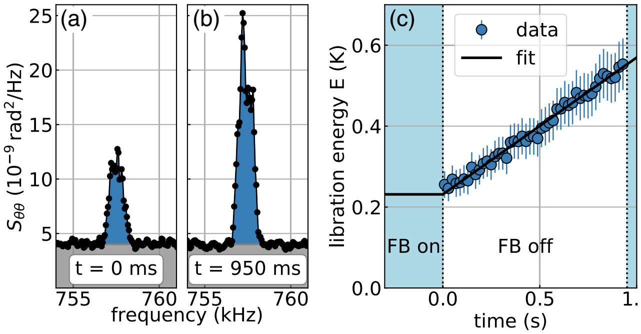

FIG. 2. Reheating experiment at $p_{\mathrm{gas}} = 1.1(1) \times 10^{-8}$ mbar. (a) Cooled libration spectrum right after feedback cooling is turned off. (b) Libration spectrum after 950 ms, just before the feedback cooling is turned back on. (c) Libration energy (blue circles) as a function of time. A linear fit to the data is shown as the solid line.

Our second cross-check of this hypothesis is by comparison of Γ_{res} to the theoretical expectation. In a simple model, treating the dumbbell as a lossless anisotropic dipolar scatterer, the heating rate Γ_{sn} expected due to radiation torque shot noise is given by [20,30–32]

$$\Gamma_{\mathrm{sn}} = \frac{1}{2}\left(\frac{\Delta\alpha}{\alpha_x}\right)^2 \hbar^2 \frac{P}{I\hbar\omega}, \qquad (3)$$

with P the power scattered by the dumbbell, α_x its polarizability along the long axis, and $\Delta\alpha$ the difference in polarizability of the long and short axes. Within this model, we estimate the expected shot noise heating rate to be $\Gamma_{\mathrm{sn}} = 0.18$ K/s, shown in Fig. 3 as the solid red line. The value for the heating rate predicted by the model matches the measured value to within a factor of 3. The discrepancy between theory and experiment can be accounted for by the accuracy of the calibration procedure. A detailed discussion

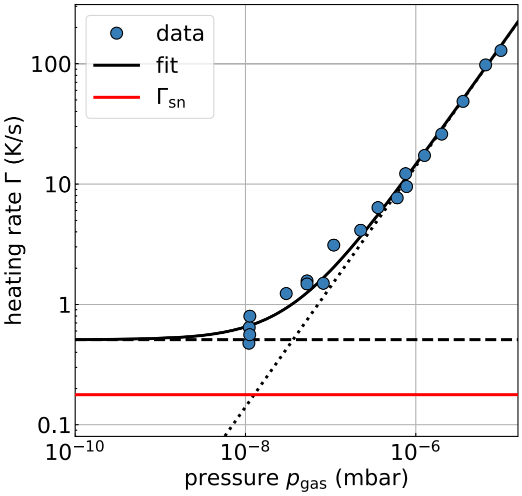

FIG. 3. Heating rate (blue circles) as a function of pressure. The solid black curve shows a linear fit with constant offset Γ_{res} (dashed line). The dotted line indicates the contribution of the gas to the heating rate. The red solid line shows the theoretical prediction for the radiation torque shot noise heating rate Γ_{sn}. See the Supplemental Material [33] for details on the model and error estimates.

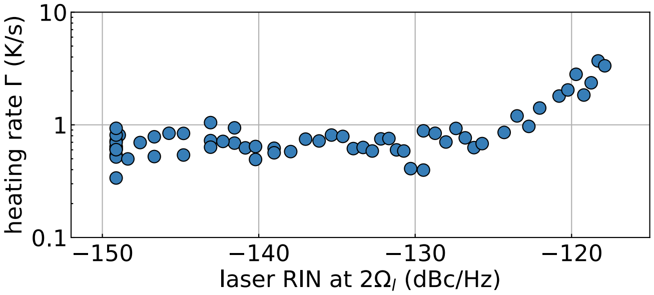

FIG. 4. Heating rate as a function of relative intensity noise (RIN) at $2\Omega_l$ at $1.1(1) \times 10^{-8}$ mbar (blue circles). An effect on the heating rate is observable only for RIN values exceeding −125 dBc/Hz.

of the model and its validity, as well as the errors, is provided in the Supplemental Material [33]. The order-of-magnitude agreement between theory and experiment supports the hypothesis that the observed heating rate arises from radiation torque shot noise.

Finally, we exclude classical laser noise as a source of the observed heating rate at low pressures. To this end, we introduce additional relative intensity noise (RIN) into the system by adding white noise with a bandwidth of 50 MHz and variable variance to the feedback signal entering the electro-optical modulator. Without added noise, our laser has a measured RIN of −146 dBc/Hz at Ω_l and −149 dBc/Hz at $2\Omega_l$. A detailed analysis of the RIN can be found in the Supplemental Material [33]. In Fig. 4, we plot the heating rate measured at a pressure of $1.1(1) \times 10^{-8}$ mbar as a function of laser RIN at $2\Omega_l$ (blue circles). The heating rate remains constant up to a RIN of −125 dBc/Hz and increases only for higher RIN values. We therefore conclude that the influence of the baseline RIN on the heating rates reported in Fig. 3 is negligible.

Conclusion.—We have cooled the librational motion of an optically levitated dumbbell to a record low temperature of 240 mK. Furthermore, we have observed signatures of radiation torque shot noise on a nanomechanical rotor for the first time. In particular, we have demonstrated that the heating rate of the librational motion of an optically levitated nanodumbbell in high vacuum is independent of the coupling to the thermal bath and of classical laser noise. Comparison to the theoretical expectation further supports the hypothesis that the observed heating rate is due to radiation torque shot noise. Entering this regime generates the opportunity to exploit engineering of the quantum properties of light—for example, by polarization squeezing [48]—to control rotational motion. Furthermore, in this backaction-limited regime, it is possible to gain measurement-based quantum control over optomechanical systems [7] with the aim to test quantum mechanical effects in rotating systems at a macroscopic scale [27,28]. Importantly, we establish parametric feedback cooling as a powerful technique to control rotational motion. Therefore, this work brings ground-state cooling and quantum control of optically levitated librators firmly within reach.

Furthermore, our work is of significance for the development of torque sensors based on levitated nanoparticles [21] with potential applications for the characterization of materials at the nanoscale [49–51] and for the detection of angular momentum states of light [52]. Our experiments constitute an important step toward operating those sensors at the standard quantum limit, which requires careful balancing of measurement backaction with intrinsic damping [14]. At this limit, levitated torque sensors hold promise to provide access to currently elusive but deeply fundamental effects of vacuum friction [53–57].

This work was supported by an ETH Research Grant (Grant No. ETH-47 20-2) and by the NCCR-QSIT program (Grant No. 51NF40-185902). We thank E. Bonvin, J. Gao, M.L. Mattana, A. Militaru, J. Piotrowski, M. Rossi, D.Windey, and J. Zielinska for valuable discussions.

---

[*] Corresponding author.
vfons@ethz.ch

[1] W. Bowen and G. Milburn, *Quantum Optomechanics* (Taylor & Francis, London, 2015).

[2] M. Aspelmeyer, T. Kippenberg, and F. Marquardt, *Cavity Optomechanics: Nano- and Micromechanical Resonators Interacting with Light*, Quantum Science and Technology (Springer Berlin Heidelberg, Berlin Heidelberg, 2014).

[3] V. B. Braginsky, Sov. Phys. JETP **26**, 831 (1968).

[4] S. Schreppler, N. Spethmann, N. Brahms, T. Botter, M. Barrios, and D. M. Stamper-Kurn, Science **344**, 1486 (2014).

[5] D. Mason, J. Chen, M. Rossi, Y. Tsaturyan, and A. Schliesser, Nat. Phys. **15**, 745 (2019).

[6] V. Sudhir, D. J. Wilson, R. Schilling, H. Schütz, S. A. Fedorov, A. H. Ghadimi, A. Nunnenkamp, and T. J. Kippenberg, Phys. Rev. X **7**, 011001 (2017).

[7] M. Rossi, D. Mason, J. Chen, Y. Tsaturyan, and A. Schliesser, Nature (London) **563**, 53 (2018).

[8] O. Romero-Isart, A. C. Pflanzer, F. Blaser, R. Kaltenbaek, N. Kiesel, M. Aspelmeyer, and J. I. Cirac, Phys. Rev. Lett. **107**, 020405 (2011).

[9] O. Romero-Isart, New J. Phys. **19**, 123029 (2017).

[10] T. P. Purdy, R. W. Peterson, and C. A. Regal, Science **339**, 801 (2013).

[11] V. Jain, J. Gieseler, C. Moritz, C. Dellago, R. Quidant, and L. Novotny, Phys. Rev. Lett. **116**, 243601 (2016).

[12] P. Lebedew, Ann. Phys. (N.Y.) **311**, 433 (1901).

[13] C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmermann, Rev. Mod. Phys. **52**, 341 (1980).

[14] P. H. Kim, B. D. Hauer, C. Doolin, F. Souris, and J. P. Davis, Nat. Commun. **7**, 13165 (2016).

[15] T. Delord, P. Huillery, L. Nicolas, and G. Hétet, Nature (London) **580**, 56 (2020).

[16] K. L. Dooley, L. Barsotti, R. X. Adhikari, M. Evans, T. T. Fricke, P. Fritschel, V. Frolov, K. Kawabe, and N. Smith-Lefebvre, J. Opt. Soc. Am. A **30**, 2618 (2013).

[17] K. Komori, Y. Enomoto, C. P. Ooi, Y. Miyazaki, N. Matsumoto, V. Sudhir, Y. Michimura, and M. Ando, Phys. Rev. A **101**, 011802(R) (2020).

[18] J. Millen, T. S. Monteiro, R. Pettit, and A. N. Vamivakas, Rep. Prog. Phys. **83**, 026401 (2020).

[19] R. Reimann, M. Doderer, E. Hebestreit, R. Diehl, M. Frimmer, D. Windey, F. Tebbenjohanns, and L. Novotny, Phys. Rev. Lett. **121**, 033602 (2018).

[20] J. Ahn, Z. Xu, J. Bang, Y. H. Deng, T. M. Hoang, Q. Han, R. M. Ma, and T. Li, Phys. Rev. Lett. **121**, 033603 (2018).

[21] J. Ahn, Z. Xu, J. Bang, P. Ju, X. Gao, and T. Li, Nat. Nanotechnol. **15**, 89 (2020).

[22] S. Kuhn, A. Kosloff, B. A. Stickler, F. Patolsky, K. Hornberger, M. Arndt, and J. Millen, Optica **4**, 356 (2017).

[23] J. Bang, T. Seberson, P. Ju, J. Ahn, Z. Xu, X. Gao, F. Robicheaux, and T. Li, Phys. Rev. Research **2**, 043054 (2020).

[24] F. Tebbenjohanns, M. Frimmer, V. Jain, D. Windey, and L. Novotny, Phys. Rev. Lett. **124**, 013603 (2020).

[25] U. Delić, M. Reisenbauer, K. Dare, D. Grass, V. Vuletić, N. Kiesel, and M. Aspelmeyer, Science **367**, 892 (2020).

[26] B. A. Stickler, S. Nimmrichter, L. Martinetz, S. Kuhn, M. Arndt, and K. Hornberger, Phys. Rev. A **94**, 033818 (2016).

[27] B. A. Stickler, B. Schrinski, and K. Hornberger, Phys. Rev. Lett. **121**, 040401 (2018).

[28] B. A. Stickler, B. Papendell, S. Kuhn, B. Schrinski, J. Millen, M. Arndt, and K. Hornberger, New J. Phys. **20**, 122001 (2018).

[29] R. A. Beth, Phys. Rev. **50**, 115 (1936).

[30] B. A. Stickler, B. Papendell, and K. Hornberger, Phys. Rev. A **94**, 033828 (2016).

[31] C. Zhong and F. Robicheaux, Phys. Rev. A **95**, 053421 (2017).

[32] T. Seberson and F. Robicheaux, Phys. Rev. A **102**, 033505 (2020).

[33] See Supplemental Material, which includes Refs. [34–43], at http://link.aps.org/supplemental/10.1103/PhysRevLett.127.123605 for the details about estimating Γ_{sn}, calibration of libration signal, and characterization of the RIN of the laser.

[34] J. Gieseler, Ph.D. thesis, Universitat Politecnica de Catalunya, 2014.

[35] E. Hebestreit, M. Frimmer, R. Reimann, C. Dellago, F. Ricci, and L. Novotny, Rev. Sci. Instrum. **89**, 033111 (2018).

[36] J. R. Taylor, *Classical Mechanics* (University Science Books, Mill Valley, California, 2005), p. 411.

[37] L. Novotny and B. Hecht, *Principles of Nano-Optics*, 2nd ed. (Cambridge University Press, Cambridge, England, 2012).

[38] H. Giesche, Surfactant Sci. Ser. **92**, 126 (2000).

[39] K. Nozawa, H. Gailhanou, L. Raison, P. Panizza, H. Ushiki, E. Sellier, J. P. Delville, and M. H. Delville, Langmuir **21**, 1516 (2005).

[40] M. Pitkonen, J. Appl. Phys. **103**, 104910 (2008).

[41] I. H. Malitson, J. Opt. Soc. Am. **55**, 1205 (1965).

[42] G. Ghosh, Opt. Commun. **163**, 95 (1999).

[43] B. N. Khlebtsov, V. A. Khanadeev, and N. G. Khlebtsov, Langmuir **24**, 8964 (2008).

[44] J. Gieseler, B. Deutsch, R. Quidant, and L. Novotny, Phys. Rev. Lett. **109**, 103603 (2012).

[45] F. van der Laan, R. Reimann, A. Militaru, F. Tebbenjohanns, D. Windey, M. Frimmer, and L. Novotny, Phys. Rev. A **102**, 013505 (2020).

[46] T. Seberson and F. Robicheaux, Phys. Rev. A **99**, 013821 (2019).

[47] F. Tebbenjohanns, M. Frimmer, A. Militaru, V. Jain, and L. Novotny, Phys. Rev. Lett. **122**, 223601 (2019).

[48] W. P. Bowen, R. Schnabel, H.-A. Bachor, and P. K. Lam, Phys. Rev. Lett. **88**, 093601 (2002).

[49] M. Wu, N. L.-Y. Wu, T. Firdous, F. Fani Sani, J. E. Losby, M. R. Freeman, and P. E. Barclay, Nat. Nanotechnol. **12**, 127 (2017).

[50] P. H. Kim, B. D. Hauer, T. J. Clark, F. Fani Sani, M. R. Freeman, and J. P. Davis, Nat. Commun. **8**, 1355 (2017).

[51] J. E. Losby, V. T. K. Sauer, and M. R. Freeman, J. Phys. D **51**, 483001 (2018).

[52] *The Angular Momentum of Light*, edited by D. Andrews and M. Babiker (Cambridge University Press, Cambridge, England, 2012).

[53] M. Kardar and R. Golestanian, Rev. Mod. Phys. **71**, 1233 (1999).

[54] A. Manjavacas and F. J. García De Abajo, Phys. Rev. Lett. **105**, 113601 (2010).

[55] R. Zhao, A. Manjavacas, F. J. García De Abajo, and J. B. Pendry, Phys. Rev. Lett. **109**, 123604 (2012).

[56] A. Manjavacas, F. J. Rodríguez-Fortuño, F. J. García De Abajo, and A. V. Zayats, Phys. Rev. Lett. **118**, 133605 (2017).

[57] Z. Xu and T. Li, Phys. Rev. A **96**, 033843 (2017).