1.

The situation described in the problem is <u>not</u> physical because it can not be set arbitrarily the radius and velocity of a particle moving in a circular orbit with the magnitude of the velocity constant.

However we can calculated the energy radiated per cycle in the context of a fictitious world for this problem.

The power radiated is

$$P = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 c}\gamma^6\left[\left(\dot{\vec{\beta}}\right)^2 - \left(\vec{\beta}\times\dot{\vec{\beta}}\right)^2\right] \qquad [1]$$

where

$$\vec{\beta} = \frac{\vec{v}}{c}$$

since $\vec{v}\perp\vec{a}$ and $\frac{v^2}{r} = a$

$$\gamma \equiv \left[\sqrt{1-\frac{v^2}{c^2}}\right]^{-1}$$

eq [1] becomes

$$P = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 c}\gamma^6\left[\frac{a^2}{c^2} - \frac{a^2}{c^2}\frac{v^2}{c^2}\right]$$

$$P = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 c}\gamma^6\frac{a^2}{c^2}\left[1-\frac{v^2}{c^2}\right]$$

$$P = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\frac{\gamma^6}{\gamma^2}\frac{a^2}{c^3} = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\gamma^4\frac{a^2}{c^3}$$

$$P = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\frac{\gamma^4 v^4}{c^3 r^2} \qquad [2]$$

the time for one cycle is

$$\tau = \frac{2\pi r}{v}$$

then the energy radiated per cycle is

$$E = P\tau = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\frac{\gamma^4 v^4}{c^3 r^2}\frac{2\pi r}{v}$$

$$\boxed{E = \frac{1}{3\epsilon_0}\frac{e^2\gamma^4}{r}\cdot\frac{v^3}{c^3} = \frac{1}{3\epsilon_0}\frac{e^2\gamma^4}{r}\beta^3}$$

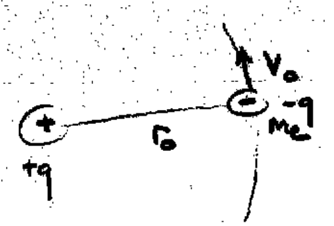
a] for $r = 100\,m$ $\quad v_0 = 0.1c$

$$\boxed{E = 1.947\times10^{-28}\,J}$$

b] for $r = 100\,m$ $\quad v_0 = 0.9c$

$$\boxed{E = 1.947\times10^{-29}\,J}$$

ESTIMATE FOR COLLAPSE-TIME OF HYDROGEN ATOM



$\varepsilon_0 = 0.885 \; 10^{-11} \; As/Vm$

$q = 1.602 \; 10^{-19} \; Cb$

$m_e = 0.991 \; 10^{-30} \; kg$

$r_0 = 0.529 \; 10^{-10} \; m$

Equation of motion (no radiation) : $\frac{1}{4\pi\varepsilon_0} \frac{q^2}{r_0^2} = m_e \frac{v_0^2}{r_0}$

$$\longrightarrow v_0 = 2.118 \; 10^6 \, m/s \quad (v_0 \ll c) \tag{1}$$

Time for one orbit : $t_0 = 2\pi r_0 / v_0 = 1.52 \; 10^{-16} \, s$

Radiation : $\frac{dE}{dt} = \frac{1}{4\pi\varepsilon_0} \frac{2q^2 a_0^2}{3c^3} \quad (v_0 \ll c)$ with $a_0 = \frac{v_0^2}{r_0}$

$$\longrightarrow \frac{dE}{dt} = \frac{1}{(4\pi\varepsilon_0)^3} \frac{2q^6}{3m_e^2 c^3} r_0^{-4} = 4.668 \; 10^{-8} \, W \tag{2}$$

Energy : $E_0 = \frac{1}{2} m v_0^2 - \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r_0} = -\frac{1}{8\pi\varepsilon_0} \frac{q^2}{r_0} = -2.18 \; 10^{-18} \, J = -13.613 \, eV$ (3)

1st estimate : Assume electron stays on same orbit. How long does it take to radiate the initial energy E_0 ?

$$\boxed{t_f = \frac{E_0}{(dE/dt)} = 0.47 \; 10^{-10} \, sec} \tag{4}$$

2nd estimate : Radius of orbit is changing slowly

From (2) → rate of energy change : $\frac{dE}{dt} = -\frac{1}{(4\pi\varepsilon_0)^3} \frac{2q^6}{3m_e^2 c^3} r^{-4}(t)$ (5)

From (3) → actual energy : $E(t) = \frac{1}{8\pi\varepsilon_0} \frac{q^2}{r(t)}$ (6)

Combining (5) + (6) : $$\boxed{\frac{dE}{dt} = -k\, E^4(t)}$$ (7

$$k = \frac{4\pi\varepsilon_0}{q^2} \frac{2}{3 m_e^2 c^3} = 1.092\, 10^{62}\, J^{-3} s^{-1} = 4.49\, 10^{5}\, eV^{-3} s^{-1}$$

Solution : $$\frac{dE}{E^4} = -k\, dt \longrightarrow \int_{E_0}^{E_f} E^{-4}\, dE = -k \int_0^{t_f} dt$$

$$\longrightarrow t_f = \frac{1}{3k} \left[\frac{1}{E_f^3} - \frac{1}{E_0^3} \right]$$ (8

Initial + Final energy: $$E_0 = -\frac{1}{4\pi\varepsilon_0} \frac{q^2}{r_0} = -13.61\ eV$$

$$E_f = -\frac{1}{4\pi\varepsilon_0} \frac{q^2}{r_e} = -2.55\, 10^{5}\, eV$$

classical electron radius : $m_e c^2 = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r_e}$

$$\longrightarrow \frac{1}{E_f^3} \approx 0$$

thus : $$\boxed{t_f = -\frac{1}{3k} \frac{1}{E_0^3} = 2.94\, 10^{-10}\ sec}$$ (9

2

angular distribution of radiated energy

$$\frac{dP(t)}{d\Omega} = \frac{e^2 a^2}{4\pi(4\pi\epsilon_0)c^3} \frac{\sin^2\theta}{(1-\frac{v}{c}\cos\theta)^5} \qquad [1]$$

The above equation is valid if $\vec{v}$ and $\vec{a}$ are in the same direction, the distance R from the particle to the observation point is very large so [illegible] and the observation unitary vector $\hat{n}$ are negl[illegible]

since the electron is desacelerated uniformly then

$$v = v_0 - at \qquad [2]$$

$$v(t_f) = 0 \Rightarrow t_f = \frac{v_0}{a} \qquad [3]$$

$$\frac{dE}{d\Omega} = \int_0^{\frac{v_0}{a}} \frac{dP}{d\Omega} dt = \frac{e^2 a^2}{4\pi(4\pi\epsilon_0)c^3} \sin^2\theta \int_0^{\frac{v_0}{a}} \frac{dt}{\left(1-\frac{(v_0-at)\cos\theta}{c}\right)^5} \qquad [4]$$

$$\frac{dE}{d\Omega} = \frac{e^2 a^2 \sin^2\theta}{4\pi(4\pi\epsilon_0)c^3 (-4)\frac{a}{c}\cos\theta} \left. \frac{1}{\left[1-\frac{(v_0-at)\cos\theta}{c}\right]^4} \right|_{t=0}^{t=\frac{v_0}{a}}$$

$$\frac{dE}{d\Omega} = \frac{e^2 a}{4\pi[4\pi\epsilon_0]c^2} \frac{\sin^2\theta}{\cos\theta} \left(\frac{1}{-4}\right) \left[1 - \frac{1}{[1-\frac{v_0}{c}\cos\theta]^4}\right]$$

$$\boxed{\frac{dE}{d\Omega} = \frac{e^2 a}{4^3\pi^2\epsilon_0 c^2} \frac{\sin^2\theta}{\cos\theta} \left[\frac{1}{[1-\frac{v_0}{c}\cos\theta]^4} - 1\right]} \qquad [5]$$

Total radiated energy

$$E = \int \frac{dE}{d\Omega} d\Omega = \int_0^{\pi}\int_0^{2\pi} \frac{e^2 a}{4^3\pi^2\epsilon_0 c^2} \frac{\sin^2\theta}{\cos\theta} \left[\frac{1}{[1-\frac{v_0}{c}\cos\theta]^4} - 1\right] d\Omega\, d\phi$$

$$\boxed{E = \frac{2e^2 a}{4^3\pi\epsilon_0 c^2} \int_0^{\pi} \frac{\sin^3\theta}{\cos\theta} \left[\frac{1}{[1-\frac{v_0}{c}\cos\theta]^4} - 1.\right] d\theta} \qquad [6]$$

$$T = m_{0e} c^2 \left[\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} - 1\right]$$

$$\frac{T}{m_{0e}c^2} + 1 = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

$$1 - \frac{v_0^2}{c^2} = \frac{1}{\left[\frac{T}{m_{0e}c^2} + 1\right]^2}$$

$$\frac{v_0}{c} = \sqrt{1 - \frac{1}{\left[\frac{T}{m_{0e}c^2}+1\right]^2}} \qquad [7]$$

For $T = 100\ \text{KeV}$ and $m_0 = 9.11 \times 10^{-31}$ kg

$$\boxed{\frac{v_0}{c} = 0.547 \qquad [8]}$$

Therefore by the factor $\frac{v_0}{c}$ into eqs [5] and [6]

we obtan that (numerical integration

$$E = \frac{2e^2 a}{4^3 \pi \epsilon_0 c^2}(4.274)$$

$$E = \frac{1}{32}\frac{e^2 a}{\pi \epsilon_0 c^2}(4.274) \qquad [9]$$

Equation [9] is valid if

$$E < T$$

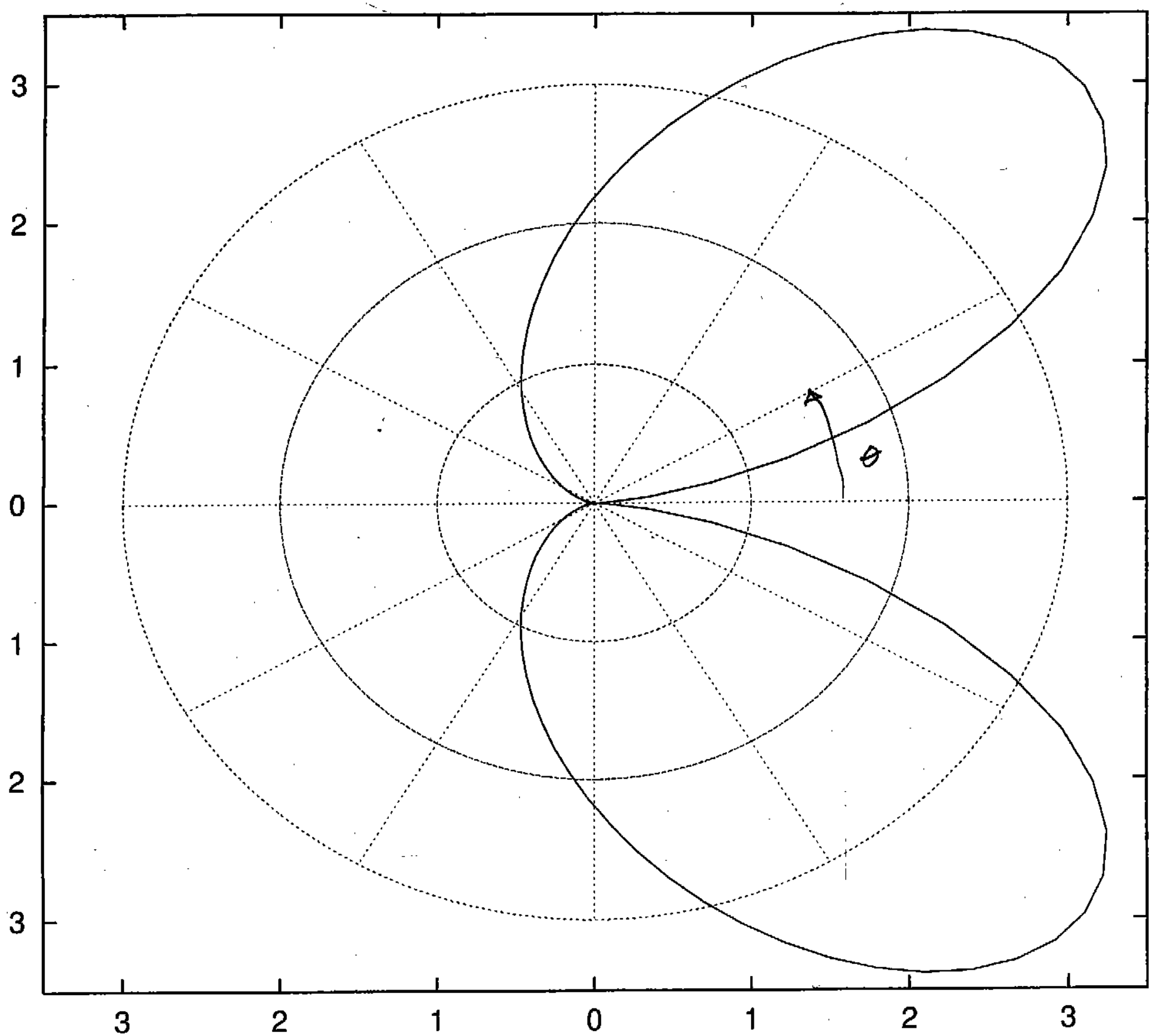
$$\frac{1}{32}\frac{e^2 a}{\pi \epsilon_0 c^2}(4.274) < T$$

$$a < \frac{32 \pi \epsilon_0 c^2 T}{e^2 (4.274)}$$

For $T = 100\ \text{KeV}$

$$a < 1.17 \times 10^{31} \frac{m}{s^2}$$

So the limit case, the particle should be in rest in a time $\approx 10^{-24}$ sec.



Polar plot of

$$\frac{dE/d\Omega(\theta)}{\frac{e^2a}{4^3\pi^2\epsilon_0 c^2}}$$