1

The field $\vec{E}$ has to fulfill the inhomogeneous equation when sources are embedded in a homogeneous space, with dielectric constant ϵ

$$\nabla^2 \vec{E}(\vec{r}) + k_0^2 \epsilon\, \vec{E}(\vec{r}) = -i\mu_0 \omega \vec{J}_e(\vec{r}) \qquad [1]$$

So the Green's function should satisfy

$$\nabla \times \nabla \times \vec{G}(\vec{r},\vec{r}') - k_0^2 \epsilon\, \overleftrightarrow{G}(\vec{r},\vec{r}') = \overleftrightarrow{I}\, \delta(\vec{r}-\vec{r}') \qquad [3]$$

where $\overleftrightarrow{I}$ is the unitary dyadic

The requirements for the solution of eq [3] is all components of $\vec{G}(\vec{r},\vec{r}')$ should <u>vanish at infinity</u> and that it satisfy the <u>radiation condition</u>

Let take the divergence of eq [3]

$$\nabla \cdot \nabla \times \left[\nabla \times \vec{G}(\vec{r},\vec{r}')\right] - k_0^2 \epsilon\, \nabla \cdot \vec{G}(\vec{r},\vec{r}') = +\nabla \delta(\vec{r}-\vec{r}') = -\nabla' \delta(\vec{r}-\vec{r}')$$

$$k_0^2 \epsilon\, \nabla \cdot \vec{G}(\vec{r},\vec{r}') = -\nabla \delta(\vec{r},\vec{r}') = \nabla' \delta(\vec{r},\vec{r}') \qquad [4]$$

using the fact that

$$\nabla \times (\nabla \times \quad) \equiv \nabla(\nabla \cdot \quad) - \nabla^2(\quad)$$

$$\nabla(\nabla \cdot \vec{G}(\vec{r},\vec{r})) - \nabla^2 \vec{G}(\vec{r},\vec{r}) - k_0^2 \epsilon\, \vec{G}(\vec{r},\vec{r}_1) = \vec{I}\, \delta(\vec{r}-\vec{r}')$$

$$(\nabla^2 + k_0^2 \epsilon)\, \vec{G}(\vec{r},\vec{r}') = -\overleftrightarrow{I}\, \delta(\vec{r}_1-\vec{r}') + \nabla(\nabla \cdot \vec{G}(\vec{r},\vec{r}')) \qquad [5]$$

From eq [4], eq [5] becomes

$$(\nabla^2 + k_0^2 \epsilon)\, \vec{G}(\vec{r},\vec{r}') = -\left\{\overleftrightarrow{I} + \frac{1}{k^2 \epsilon} \nabla \nabla \cdot \right\} \delta(\vec{r}-\vec{r}') \qquad [6]$$

A solution for eq [6] is

$$\boxed{\overleftrightarrow{G}(\vec{r},\vec{r}') = \left(I + \frac{1}{k_0^2 \epsilon} \nabla \nabla \cdot\right) G_0(\vec{r},\vec{r}')} \qquad [7]$$

if $G_0(\vec{r},\vec{r}')$ satisfy

$$\boxed{(\nabla^2 + k_0^2 \epsilon)\, G_0(\vec{r},\vec{r}') = -\delta(\vec{r}-\vec{r}')} \qquad [8]$$

Then the solution for eq [8] that satisfy the radiation condition and that it vanishes at infinity is

$$\boxed{G_0(\vec{r},\vec{r}') = \frac{e^{i k_0 \sqrt{\epsilon}\, |\vec{r}-\vec{r}'|}}{4\pi\, |\vec{r}-\vec{r}'|}} \qquad [9]$$

Substituting eq [9] into eq [8]

$$\vec{\vec{G}}_r(\vec{r},\vec{r}\,') = -\left\{ \vec{\vec{I}} - \frac{(1 - i k_r R)}{k_r^2 R^2}\vec{\vec{I}} - \frac{\{-3 + 3i k_r R + k_r^2 R^2\}\vec{R}\vec{R}}{k_r^2 R^4} \right\} \times \frac{e^{i k_r R}}{4\pi R} \qquad [10]$$

where $R = |\vec{r} - \vec{r}\,'|$ $k_r^2 \equiv k_0^2 \epsilon$ and

$\vec{R}\vec{R}$ is the exterior product that gives a dyadics, i.e.

$$\vec{R}\vec{R} = \sum_{i=1}^{3}\sum_{j=1}^{3} x_i x_j \hat{x}_i \hat{x}_j \qquad [11]$$

Now we split

$$\vec{\vec{G}}_r(\vec{r},\vec{r}\,') = \vec{\vec{G}}_{nf}(\vec{r},\vec{r}\,') + \vec{\vec{G}}_{if}(\vec{r},\vec{r}\,') + \vec{\vec{G}}_{ff}(\vec{r},\vec{r}\,') \qquad [12]$$

nf (nearfield)

if (intermediate field)

ff (far field)

where

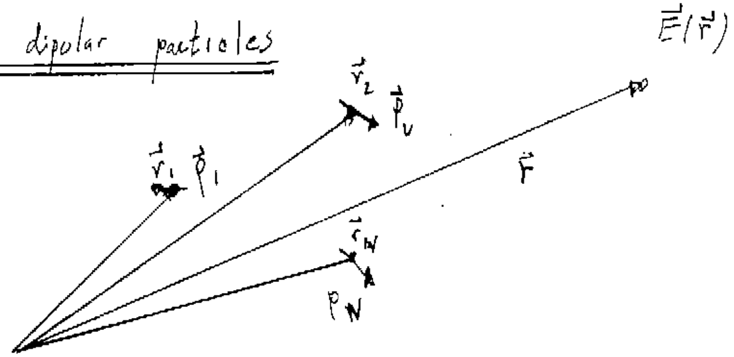
$$\vec{\vec{G}}_{nf}(\vec{r},\vec{r}\,') = \left[\frac{1}{k_r^2 R^2}\vec{\vec{I}} - \frac{3\vec{R}\vec{R}}{R^4 k_r^2} \right] \frac{e^{i k_r R}}{4\pi R} \qquad [13]$$

$$\vec{\vec{G}}_{ni}(\vec{r},\vec{r}\,') = \left[\frac{-i}{k_r R}\vec{\vec{I}} + \frac{3i}{k_r R^3}\vec{R}\vec{R} \right] \frac{e^{i k_r R}}{4\pi R} \qquad [14]$$

$$\vec{\vec{G}}_{ff}(\vec{r},\vec{r}\,') = \left[-\vec{\vec{I}} + \frac{\vec{R}\vec{R}}{R^2} \right] \frac{e^{i k_r R}}{4\pi R} \qquad [15]$$

2

a] N dipolar particles



The Maxwell's equation for the electric field $\vec{E}(\vec{r})$ is given by

$$\nabla \times \nabla \times \vec{E} - k_0^2 E(\vec{r}) = i\omega\mu_0 J_e(\vec{r}) \qquad [1]$$

In this case

$$J_e(\vec{r}) = \sum_{n=1}^{N} (-i\omega)\vec{p}_n \delta(\vec{r}-\vec{r}_n) \qquad [2]$$

Using the Green's formalism the solution of eq. [1] is

$$\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + i\omega\mu_0 \int_V dv' \overleftrightarrow{G}(\vec{r},\vec{r}')\cdot\vec{J}_e(\vec{r}') \qquad r \in V$$

where $\vec{E}_0(\vec{r})$ is the homogeneous solution of eq [1] and $\overleftrightarrow{G}(\vec{r},\vec{r}')$ is the Green's function for a homogeneous medium

$$\overleftrightarrow{G}(\vec{r},\vec{r}') = \left[\overleftrightarrow{I} + \frac{1}{k^2}\nabla\nabla\right] G_0(\vec{r},\vec{r}') \qquad [3]$$

and

$$G(\vec{r},\vec{r}') \equiv \frac{1}{4\pi}\frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \qquad [4]$$

then

$$\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + i\omega\mu_0(-i\omega)\int \sum_{n=1}^{N} \overleftrightarrow{G}(\vec{r},\vec{r}')\vec{p}_n \delta(\vec{r}'-\vec{r}_n)\, d^3r'$$

$$\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + \frac{\omega^2\mu_0\epsilon_0}{\epsilon_0}\sum_{n=1}^{N}\overleftrightarrow{G}(\vec{r},\vec{r}_n)\cdot\vec{p}_n$$

$$\boxed{\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + \frac{k^2}{\epsilon_0}\sum_{n=1}^{N}\overleftrightarrow{G}(\vec{r},\vec{r}_n)\cdot\vec{p}_n \qquad \vec{r}\neq\vec{r}_n \quad n=1,2,\ldots N} \qquad [5]$$

In matrix form

$$\begin{bmatrix} E_x(\vec{r}) \\ E_y(\vec{r}) \\ E_z(\vec{r}) \end{bmatrix} = \sum_{n=1}^{N} \begin{bmatrix} G_{xx}(\vec{r},\vec{r}_n) & G_{xy}(\vec{r},\vec{r}_n) & G_{xz}(\vec{r},\vec{r}_n) \\ G_{yx}(\vec{r},\vec{r}_n) & G_{yy}(\vec{r},\vec{r}_n) & G_{yz}(\vec{r},\vec{r}_n) \\ G_{zx}(\vec{r},\vec{r}_n) & G_{zy}(\vec{r},\vec{r}_n) & G_{zz}(\vec{r},\vec{r}_n) \end{bmatrix} \begin{bmatrix} p_{xn} \\ p_{yn} \\ p_{zn} \end{bmatrix} + \begin{bmatrix} E_{0x}(\vec{r}) \\ E_{0y}(\vec{r}) \\ E_{0z}(\vec{r}) \end{bmatrix}$$

$\vec{r}\neq\vec{r}_n;\ n=1..N$

[6]

[4] N particles with "no" permanent dipole moment

To find the electric field at point $\vec r_n$, that is where the n particle is situated, we must exclude its contribution, so from eq [5]

$$\vec E(\vec r_n) = \vec E_0(\vec r_n) + \frac{k^2}{\epsilon_0}\sum_{\substack{j=1\\ n\neq j}}^{N} \overleftrightarrow{G}(\vec r_n, \vec r_j)\cdot \vec P_j \qquad [7]$$

for $n = 1, \ldots, N$

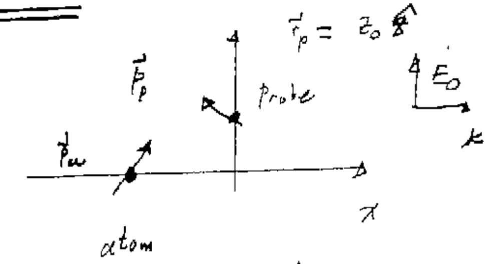
since $p_j = \alpha_j \vec E(\vec r_n)$ therefore

$$\vec E(\vec r_n) = \vec E_0(\vec r_n) + \frac{k^2}{\epsilon_0}\sum_{\substack{j=1\\ n\neq j}} \overleftrightarrow{G}(\vec r_n, \vec r_j)\,\alpha_j\,\vec E(\vec r_j) \qquad [8]$$

$$\boxed{\vec E(\vec r_n) = \vec E_0(\vec r_n) + \frac{k^2}{\epsilon_0}\sum_{\substack{j=1\\ j\neq n}} \alpha_j\,\overleftrightarrow{G}(\vec r_n, \vec r_j)\,\vec E(\vec r_j) \qquad n = 1 \ldots N \qquad [9]}$$

Therefore we have to solve a linear system of $3XN$ linear equations. Once the fields $\vec E(\vec r_n)$ are calculated $\vec P_n = \alpha_n \vec E(\vec r_n)$

-a] Two particles



$\vec r_\alpha = x\hat x$

We assume that only $\vec E_0$ drives the probe, then from eq [8]

$$\vec E(\vec r_p) = \vec E_0(\vec r_p) + \frac{k^2}{\epsilon_0}\alpha_\alpha\,\overleftrightarrow{G}(\vec r_p, \vec r_\alpha)\,\vec E(\vec r_\alpha)$$

$$\vec E(\vec r_\alpha) = \frac{k^2}{\epsilon_0}\alpha_p\,\overleftrightarrow{G}(\vec r_\alpha, \vec r_p)\,\vec E(\vec r_p) \qquad [10]$$

The above system can be written as:

$$\begin{bmatrix} 1 & 0 & 0 & -\beta_\alpha G^{p\alpha}_{xx} & -\beta_\alpha G^{p\alpha}_{xy} & -\beta_\alpha G^{p\alpha}_{xz} \\ 0 & 1 & 0 & -\beta_\alpha G^{p\alpha}_{yx} & -\beta_\alpha G^{p\alpha}_{yy} & -\beta_\alpha G^{p\alpha}_{yz} \\ 0 & 0 & 1 & -\beta_\alpha G^{p\alpha}_{zx} & -\beta_\alpha G^{p\alpha}_{zy} & -\beta_\alpha G^{p\alpha}_{zz} \\ -\beta_p G^{\alpha p}_{xx} & -\beta_p G^{\alpha p}_{xy} & -\beta_p G^{\alpha p}_{xz} & 1 & 0 & 0 \\ -\beta_p G^{\alpha p}_{yx} & -\beta_p G^{\alpha p}_{yy} & -\beta_p G^{\alpha p}_{yz} & 0 & 1 & 0 \\ -\beta_p G^{\alpha p}_{zx} & -\beta_p G^{\alpha p}_{zy} & -\beta_p G^{\alpha p}_{zz} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_x(\vec r_p) \\ E_y(\vec r_p) \\ E_z(\vec r_p) \\ E_x(\vec r_\alpha) \\ E_y(\vec r_\alpha) \\ E_z(\vec r_\alpha) \end{bmatrix} = \begin{bmatrix} E_{0x}(\vec r_p) \\ E_{0y}(\vec r_p) \\ E_z(\vec r_p) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Here we have define

$$\beta_{\alpha,p} \equiv \frac{k^2}{\epsilon_0}\alpha_{\alpha,p}$$

For each position point of the atom a 6 equations linear system must be solved, and the dipole strength of the dipoles is calculated

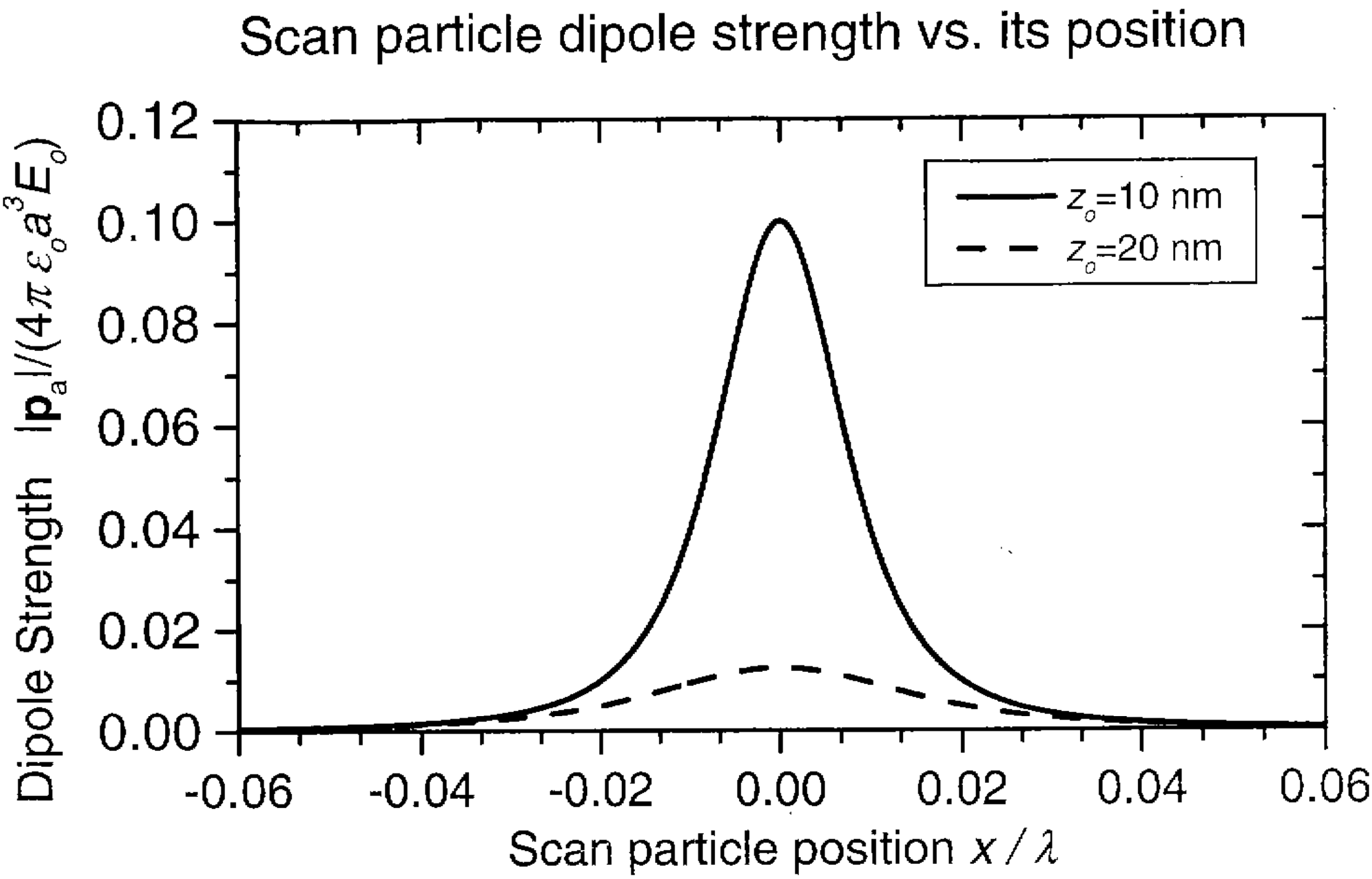
$$|\vec{p}_{a,p}|^2 = \sum_{i=1}^{3} \alpha_{a,p} \alpha_{a,p}^* E_{i(a,p)}(\vec{r}_{a,p}) E_{i(a,p)}^*(\vec{r}_{a,p})$$

In fig [1] the dipole strength of the atom as a function of the position of the atom for probe height of $z_0 = 10nm$, $20nm$, $50nm$ and $100\,nm$, is plotted

In fig [2] we show the probe dipole strength as a function of the atom position. for probe height of $z_0 = 10\,nm$, $20\,nm$, $50\,nm$ and $100\,nm$

6

FIGURE 1


Scan particle dipole strength vs. its position
Dipole Strength $|\mathbf{p}_a|/(4\pi\varepsilon_o a^3 E_o)$
Scan particle position x/λ
z_o=10 nm
z_o=20 nm
0.12
0.10
0.08
0.06
0.04
0.02
0.00
-0.06
-0.04
-0.02
0.00
0.02
0.04
0.06

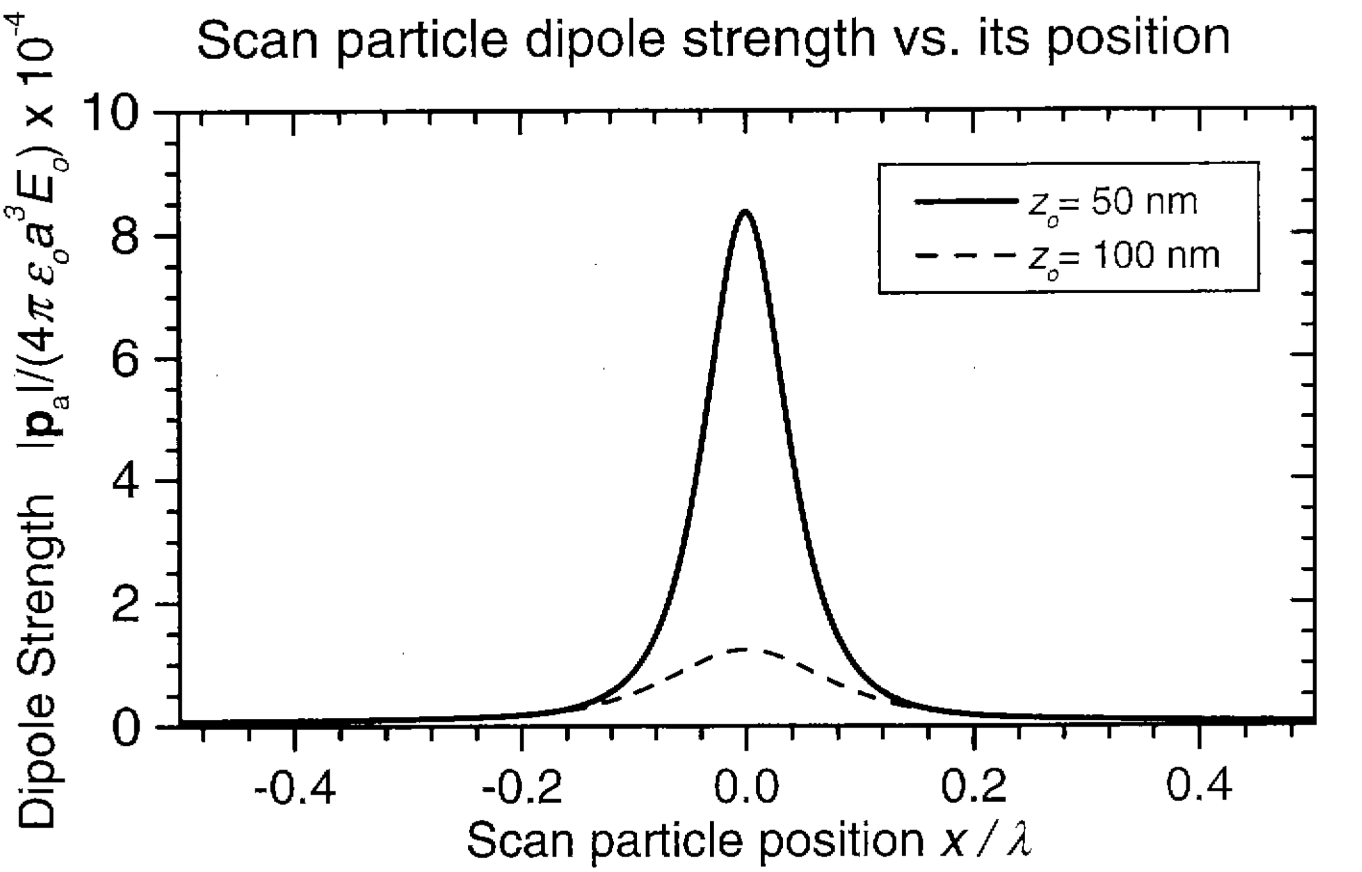
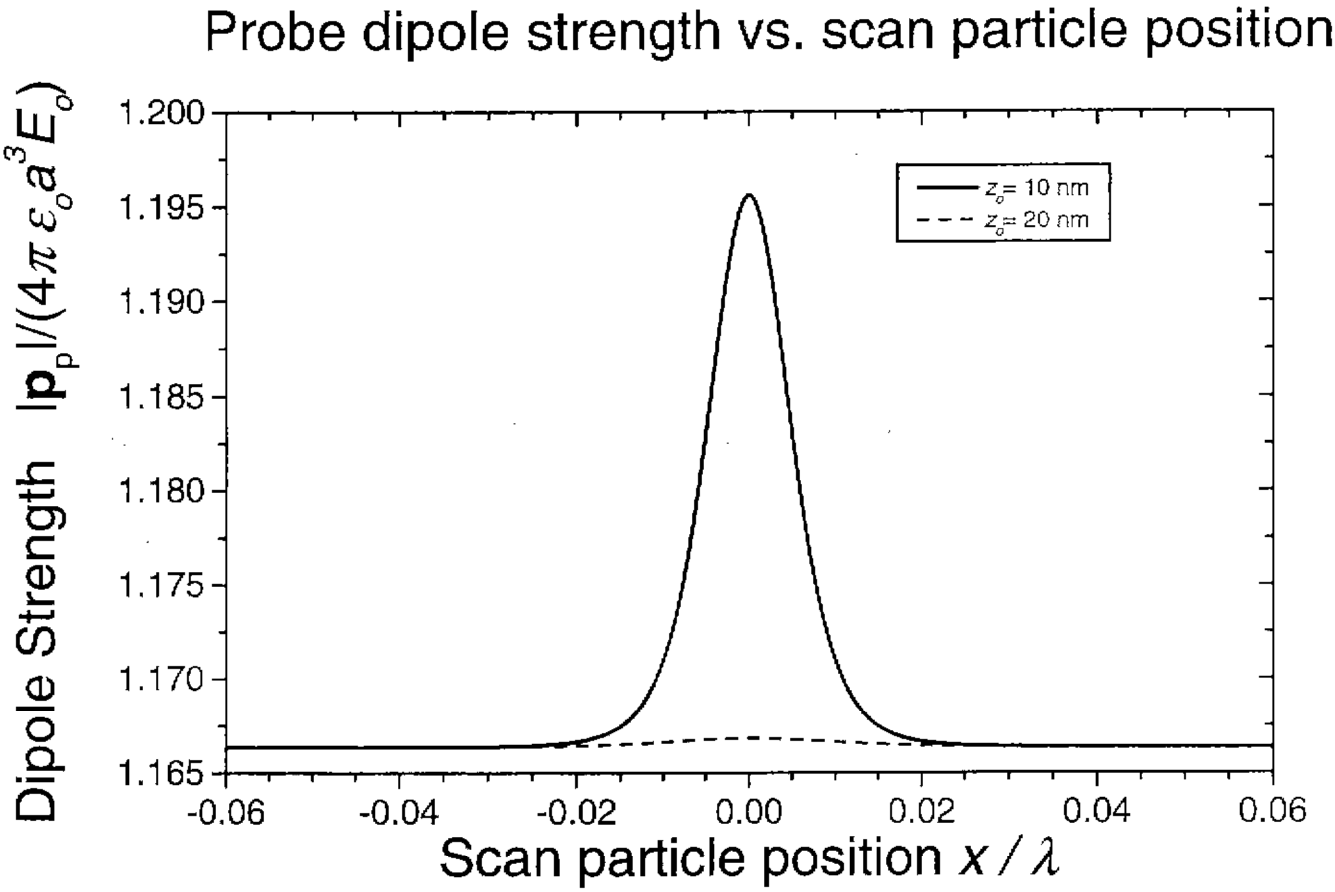
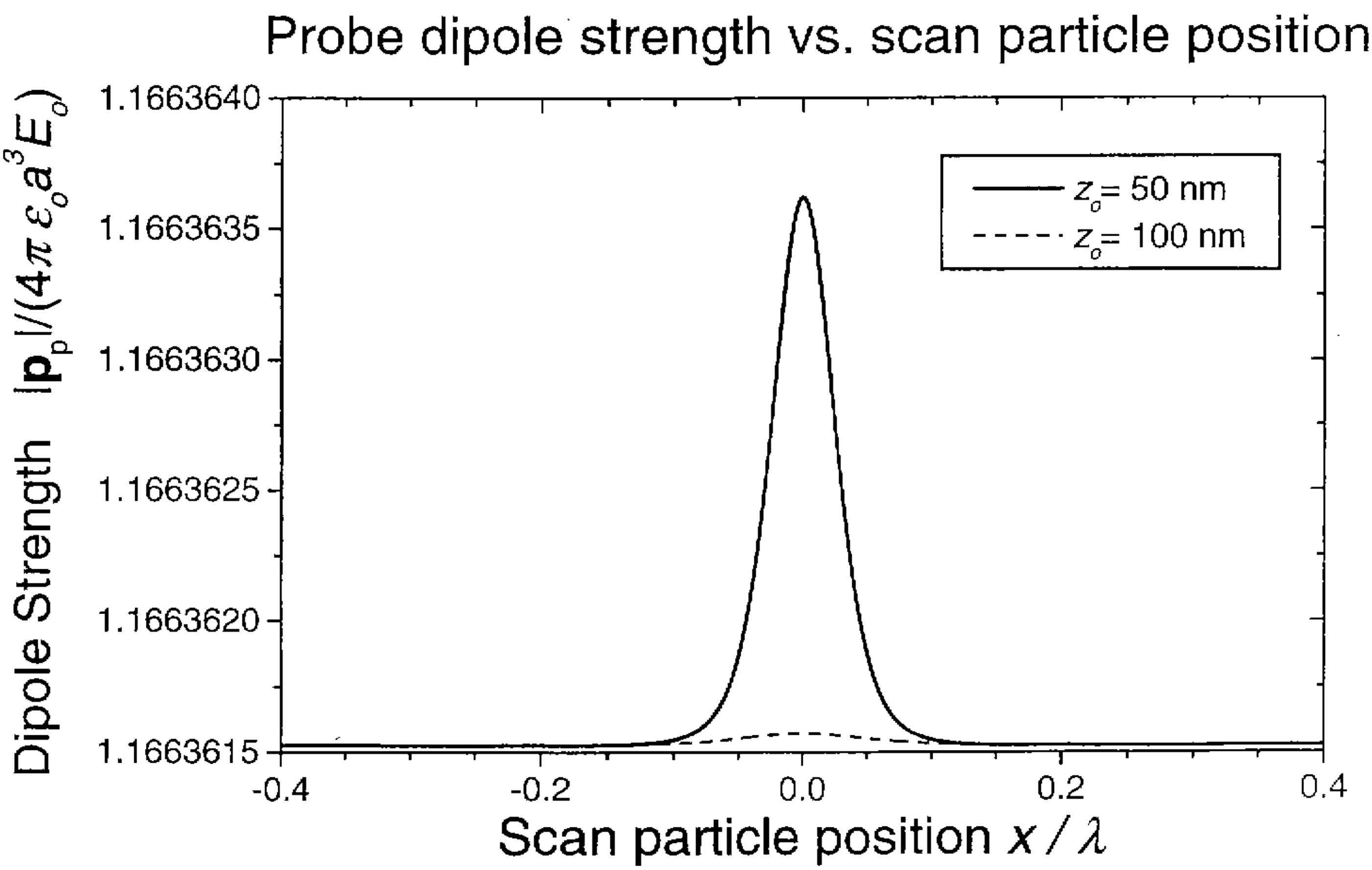

Scan particle dipole strength vs. its position
Dipole Strength $|\mathbf{p}_a|/(4\pi\varepsilon_o a^3 E_o) \times 10^{-4}$
Scan particle position x/λ
z_o= 50 nm
z_o= 100 nm
10
8
6
4
2
0
-0.4
-0.2
0.0
0.2
0.4

FIGURE 2

7


Probe dipole strength vs. scan particle position
Dipole Strength $|\mathbf{p}_p|/(4\pi\varepsilon_o a^3 E_o)$
1.200
1.195
1.190
1.185
1.180
1.175
1.170
1.165
-0.06
-0.04
-0.02
0.00
0.02
0.04
0.06
Scan particle position x/λ
z_o= 10 nm
z_o= 20 nm


Probe dipole strength vs. scan particle position
Dipole Strength $|\mathbf{p}_p|/(4\pi\varepsilon_o a^3 E_o)$
1.1663640
1.1663635
1.1663630
1.1663625
1.1663620
1.1663615
-0.4
-0.2
0.0
0.2
0.4
Scan particle position x/λ
z_o= 50 nm
z_o= 100 nm