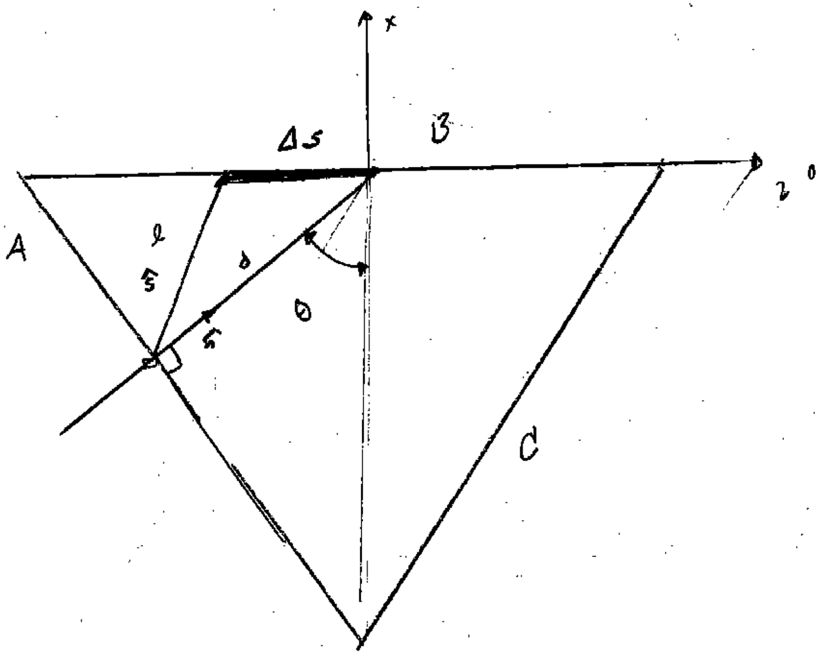
1



Since the wavevector of the incident wave is normal to the surface A the direction of the the transmitted "wavevectors for the "o wave" and "e-wave" are the same

$$\hat{n}_e = \hat{n}_o = \hat{n}$$

$$\hat{n} = \cos\theta\,\hat{x} + \sin\theta\,\hat{z} \qquad [1]$$

For the "o-wave" the direction of the Poynting vector $\hat{s}_o$ and the wave are the same then

$$\hat{s}_o = \hat{n}$$

For the "e wave" the wavevector and the Poynting vector direction are not the same, namely

$$\hat{n} \neq \hat{s}_e$$

For the e-wave, the displacement vector $\vec{D}_e$ is

$$\vec{D} = D_o\, e^{ik_o n_e(\theta)[\cos\theta\, x + \sin\theta\, y]} [\sin\theta\,\hat{x} - \cos\theta\,\hat{z}]$$

$$\vec{H}_e = \frac{\hat{n}}{n_e(\theta)} \times \vec{D} = \frac{1}{n_e(\theta)} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cos\theta & 0 & \sin\theta \\ \sin\theta & 0 & -\cos\theta \end{vmatrix} D_o\, e^{ik_o n_e(\theta)[\cos\theta x + \sin\theta y]}$$

$$\vec{E}_e = \left\{ \frac{D_o}{n_o^2}\sin\theta\,\hat{x} - \frac{D_o}{n_e^2}\cos\theta \right\} e^{ik_o n_e(\theta)[\cos\theta x + \sin\theta y]}$$

$$\langle \vec{S}_e \rangle = \frac{\vec{E}_e \times \vec{H}_e^*}{2}$$

$$\langle \vec{S}_e \rangle = \frac{1}{2} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{D_o \sin\theta}{n_o^2} & 0 & \frac{-D_o\cos\theta}{n_e^2} \\ 0 & \frac{D_o}{n_e(\theta)} & 0 \end{vmatrix}$$

$$\langle S_e \rangle = \frac{1}{2}\frac{D_o^2}{n_e(\theta)} \left[\frac{\cos\theta}{n_e^2}\hat{x} + \frac{\sin\theta}{n_o^2}\hat{z} \right]$$

Normalizing $\langle \hat{S}_e \rangle$

$$\hat{s}_e = \frac{\langle \hat{S}_e \rangle}{|\langle S_e \rangle|} = \frac{\frac{\cos\theta}{n_e^2}\,\hat{x} + \frac{\sin\theta}{n_o^2}\,\hat{z}}{\sqrt{\frac{\cos^2\theta}{n_e^4} + \frac{\sin^2\theta}{n_o^2}}} \qquad [2]$$

we have that

$$l\hat{s} = d\,\hat{n} - \Delta s\,\hat{z}$$

$$l(s_x\hat{x} + s_z\hat{z}) = d(n_x\hat{x} + n_z\hat{z}) - \Delta s\,\hat{z}$$

$$l s_x = d n_x$$

$$l s_z = d n_z - \Delta s$$

$$\frac{d n_x}{s_x} s_z = d n_z - \Delta s$$

$$\Delta s = d\left[n_z - n_x\left(\frac{s_z}{s_x}\right)\right] \qquad [3]$$

From eqs [1] and [2] eq [3] becomes

$$\Delta s = d\left[\sin\theta - \cos\theta\,\frac{n_e^2}{n_o^2}\,\frac{\sin\theta}{\cos\theta}\right]$$

$$\boxed{\Delta s = d\sin\theta\left[1 - \frac{n_e^2}{n_o^2}\right]}$$

2

The critical angle for the o-wave

$$n_o \sin\theta_{c_o} = 1$$

$$\sin\theta_{o_c} = 1/n_o = \frac{1}{1.9}$$

$$\boxed{\theta_{o_c} = 31.757^\circ}$$

For the e-wave

$$n_e(\theta_c)\sin\theta_{c_e} = 1$$

$$\sin^2\theta_{c_e} = \frac{1}{n_e^2(\theta_{c_e})} = \frac{\sin^2\theta_{c_e}}{n_o^2} + \frac{\cos^2\theta_{c_e}}{n_e^2}$$

$$\sin^2\theta_{c_e}\left[\frac{n_o^2 - 1}{n_o^2}\right] = \frac{\cos^2\theta_{c_e}}{n_e^2}$$

$$tg\,\theta_{c_e} = \frac{n_o}{n_e}\,\frac{1}{\sqrt{n_o^2 - 1}}$$

then $\boxed{\theta_{c_e} = 40.032^\circ}$

then the region for both waves undergoing internal reflection

$$R = \{ [\theta > \theta_{c_o}] \cap [\theta > \theta_{c_e}] \}$$

then

$$R = \{ \theta > \theta_{c_e} \} \Rightarrow \theta > 40.032^0$$

3

Since, the e-wave in this particular case is p-polarized then we look for the Brewster angle

$$n_e(\theta) \sin \theta_B = \sin [\theta_{T_B}]$$

$$\theta_{T_B} + \theta_B = \pi/2 \qquad n_e(\theta_B) \sin \theta_B = \cos \theta_B$$

$$tg\, \theta_B = \frac{1}{n_e(\theta_B)} \Rightarrow tg^2 \theta_B = \frac{1}{n_e^2(\theta_B)}$$

$$tg^2 \theta_B = \frac{\sin^2 \theta_B}{n_o^2} + \frac{\cos^2 \theta_B}{n_e^2}$$

$$\frac{\sin^2 \theta_B}{\cos^2 \theta_B} = \frac{1}{n_o^2} \sin^2 \theta_B + \frac{1}{n_e^2} \cos^2 \theta_B$$

$$\sin^2 \theta_B = \frac{1}{n_o^2} \sin^2 \theta_B \cos^2 \theta_B + \frac{1}{n_e^2} \cos^4 \theta_B$$

$$1 - \cos^2 \theta = \frac{1}{n_o^2} (1 - \cos^2 \theta_B) \cos^2 \theta_B + \frac{1}{n_e^2} \cos^4 \theta_B$$

$$\cos^4 \theta_B \left[\frac{1}{n_e^2} - \frac{1}{n_o^2} \right] + \cos^2 \theta_B \left[1 + \frac{1}{n_o^2} \right] - 1 = 0$$

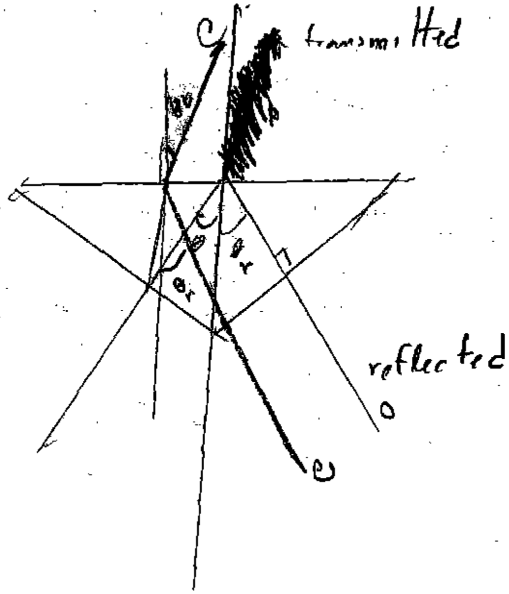
$$\cos \theta_B = 0.833$$

$$\boxed{\theta_B = 33.529^0}$$

since $\theta_B > \theta_{o_c}$

then for $\theta_B = 33.529^0$
the e-wave is totally transmitted and the o-wave totally reflected.

4 h



$\sin\theta_t = n_e(\theta_i)\sin\theta_i$

$\sin\theta_t = 35\left[\dfrac{\sin^2\theta_i}{n_o^2} + \dfrac{\cos^2\theta_i}{n_e^2}\right]^{-1/2}\sin\theta_i$

if $\theta_i = 35^\circ$

$\boxed{\theta_t = 60.59^\circ}$

For the reflected ray

$\boxed{\theta_r = \theta_i = 35^\circ}$