OPT 462: Midterm Exam March 14, 2001

(Max. points = 120. No more than 100 points required for grade **A**)

1 Attenuation of a laser pulse by a dispersive medium

At a fixed position $\mathbf{r}=0$ in space the electric field of a laser pulse is given by

$$\mathbf{E}(\mathbf{r}=0,t) \;=\; E_o \cos\omega_o t \,\frac{\sin\Delta\omega\, t}{\Delta\omega\, t}\,\mathbf{n}_x \;, \tag{1}$$

where $\omega_o \gg \Delta\omega$. The pulse propagates in free space in the direction of the positive z-axis and its field does not depend on the coordinates x and y.

- (10 points) Determine the field of the pulse anywhere in space, i.e. $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(z,t)$. What conditions have to be fulfilled in order that $\mathbf{E}(z,t)$ is a solution of Maxwell's equations?
- (10 points) Determine the spectrum $\hat{\mathbf{E}}(z,\omega)$ of the pulse and provide a graphical sketch.
- (5 points) What differential equation does $\hat{\mathbf{E}}(z,\omega)$ have to satisfy in free space? What is the corresponding equation in a dispersive medium with dielectric constant $\varepsilon(\omega)$?

The pulse is incident from the half-space $z < 0$ (vacuum) on the half-space $z > 0$ which is filled with a dispersive medium characterized by $\varepsilon(\omega)$.

- (10 points) Write down the boundary conditions at $z=0$ and determine the reflection coefficient $r(\omega)$ and the transmission coefficient $t(\omega)$ in frequency domain.
- (15 points) Determine an equation for the transmitted field $\mathbf{E}_t(\mathbf{r},t)$. Simplify as much as possible.

Hint:

$$\int_{-\infty}^{\infty} \frac{\sin(bx)}{(bx)} \exp[iax]dx \;=\; \frac{\pi}{2b}\left[sign(a+b) - sign(a-b)\right] \;. \tag{2}$$

2 Acceleration of Gallium ion

A Gallium ion (Ga^{+}) (m_o, $q=e$) is uniformly accelerated from rest to a speed v_o. The acceleration distance Δs is much smaller than the distance to the observation point, i.e. $R=|\mathbf{r}-\mathbf{r}'|$ is the same for all points on Δs.

- (10 points) How much energy is needed in order to bring the ion to its final speed?
- (20 points) What is the ratio of forward to backward radiated energy. Simplify the expression as much as possible.

3 Light scattering by a small particle near a dielectric half-space

A small particle (radius a, ε_p) is located in air at the position $(x, y, z) = (0, 0, z_o)$ above the surface $z = 0$ of a dielectric medium (ε). A plane wave with electric field amplitude E_i and wavelength λ irradiates the surface from the air-side at normal incidence.

- (5 points) Determine the field $\mathbf{E}_o$ at $(x, y, z) = (0, 0, z_o)$ in the absence of the particle.
- (5 points) What is the particle's polarizability α in the nonretarded limit?
- (25 points) The particle's induced dipole moment $\mathbf{p}$ is determined by the exciting field

  $$\mathbf{E}_{exc} \;=\; \mathbf{E}_o + \mathbf{E}_{int} \;, \tag{3}$$

  where $\mathbf{E}_{int}$ is the field due to the interaction between the particle and the surface. In the quasi-static limit ($z_o \ll \lambda$) this interaction is accounted for by replacing the half-space by an image dipole with moment

  $$\mathbf{p}_{im} = -\frac{\varepsilon - 1}{\varepsilon + 1}\,\mathbf{p} \tag{4}$$

  located at $(x, y, z) = (0, 0, -z_o)$ beneath the interface. This dipole determines the field $\mathbf{E}_{int}$. Use the free space dyadic Green's function $\overleftrightarrow{\mathbf{G}}$ to determine the particle's induced dipole moment $\mathbf{p}$. Neglect the intermediate field terms and the farfield terms. Write the result in the form

  $$\mathbf{p} \;=\; \overleftrightarrow{\alpha}_{eff}\; \mathbf{E}_o \tag{5}$$

  with $\overleftrightarrow{\alpha}_{eff}$ being the effective polarizability. Write $\overleftrightarrow{\alpha}_{eff}$ as simple as possible.
- (5 points) What is the total scattered power?