OPT 462: Homework Set 8   Due on April 4, 2001

1 Conductivity in Metals

In a metallic conductor, each atom contributes an electron into the unfilled conduction band. The conduction electrons can be treated as a free gas. Therefore, the spring constant in the Lorentz atom model is equal to zero. The conduction electrons experience damping through collisions with the lattice. The resulting drag-force of an electron is given by

$$\mathbf{F}_d = -\frac{m}{\tau}\frac{d}{dt}\mathbf{r}\,, \tag{1}$$

where m is the electron mass and τ the average collision time. An individual electron is driven by the time-dependent field $\mathbf{E}(t)$.

- Write down the equation of motion for an electron in the metal. Assume a time-harmonic field $\mathbf{E}(t)$ and solve for the drift velocity $\mathbf{v} = d\mathbf{r}/dt$.
- What is the conductivity $\sigma(\omega)$ of the metal?
- Determine the dielectric constant $\varepsilon(\omega)$ (plasma dispersion law) and compare it with $\sigma(\omega)$.
- For wavelengths shorter than $\lambda \approx 800nm$, the dielectric constant of aluminum is very well described by the plasma dispersion law. The electron density is given by $N = 175.7nm^{-3}$ and the average collision time is $\tau = 1.08fs$. Plot the dielectric constant of aluminum in the range $\lambda = [0\,..\,1000nm]$. What is the value at $\lambda = 633nm$?
- A plane wave with wavelength $\lambda = 633nm$ hits the surface of an aluminum half-space at normal incidence. Determine the skin-depth δ for which the intensity of the wave drops to $1/e$ of its value on the interface. Plot the electric field amplitude $E(z)$ inside the metal as a function of the normal coordinate z.
- Repeat the last step for a wavelength of $\lambda = 100nm$.

2 Anisotropic Crystals

Someone gives you an unidentified crystal. You go to the laboratory and you determine the following dielectric tensor

$$\overset{\leftrightarrow}{\varepsilon} = \frac{1}{16}\begin{bmatrix} 29 & \sqrt{3} & 6 \\ \sqrt{3} & 31 & -2\sqrt{3} \\ 6 & -2\sqrt{3} & 20 \end{bmatrix}\,. \tag{2}$$

- Determine whether this is an isotropic, uniaxial, or biaxial crystal.
- Consider a plane wave which propagates along any principal axis of the crystal. This plane wave has to satisfy the homogeneous Helmholtz equation and the divergence equation. What are the resulting conditions for the electric field amplitude?