OPT 462: Homework Set 7 Due on March 28, 2001

1 Light scattering by a small particle near a conducting half-space

A small silver particle (radius a, $\varepsilon(\omega)$) is located in air at the position $(x, y, z) = (0, 0, z_o)$ above the surface $z = 0$ of a silver substrate ($\varepsilon(\omega)$). A p-polarized plane wave with electric field amplitude E_i and wavelength λ irradiates the surface from the air-side at an arbitrary angle of incidence α.

- Determine the field $\mathbf{E}_o$ at $(x, y, z) = (0, 0, z_o)$ in the absence of the particle.
- What is the particle's polarizability α in the nonretarded limit? In the nonretarded (quasi-static) limit the Helmholtz equation reduces to the Laplace equation and the exciting field $\mathbf{E}_{exc}$ is uniform over the dimensions of the spherical particle. You may consult textbooks.
- The particle's induced dipole moment $\mathbf{p}$ is determined by the exciting field

$$\mathbf{E}_{exc} \;=\; \mathbf{E}_o + \mathbf{E}_{int} \;, \tag{1}$$

where $\mathbf{E}_{int}$ is the field due to the interaction between the particle and the surface. In the quasi-static limit ($z_o \ll \lambda$) this interaction is accounted for by replacing the half-space by an image dipole with moment $\mathbf{p}_{im}$ located at $(x, y, z) = (0, 0, -z_o)$ beneath the interface. Show that if $\mathbf{p}$ is parallel to the surface of the half-space the boundary conditions at the surface are fulfilled if we choose

$$\mathbf{p}_{im} = -\frac{\varepsilon - 1}{\varepsilon + 1}\,\mathbf{p}\;. \tag{2}$$

Hint: In the nonretarded limit you only consider the dipole's near-field. Furthermore, since $z_o \ll \lambda$ you can set $\exp(2ikz_o) \approx 1$.

- Determine $\mathbf{p}_{im}$ for a dipole $\mathbf{p}$ perpendicular to the interface.
- The image dipole $\mathbf{p}_{im}$ determines the field $\mathbf{E}_{int}$. Use the free space dyadic Green's function $\overleftrightarrow{\mathbf{G}}$ to determine the particle's induced dipole moment $\mathbf{p}$. Neglect the intermediate field terms and the farfield terms. Write the result in the form

$$\mathbf{p} \;=\; \overleftrightarrow{\alpha}_{eff}\;\mathbf{E}_o \tag{3}$$

with $\overleftrightarrow{\alpha}_{eff}$ being the effective polarizability. Write $\overleftrightarrow{\alpha}_{eff}$ as simple as possible.

- Arrangements of noble metal structures can lead to collective electron resonances. For the present configuration, find the distances $z_o(\varepsilon, a)$ for which these resonances occur ($\mathbf{p} \to \infty$).
- The angle of incidence is $\alpha = 45°$ and the wavelength is $\lambda = 345\,nm$. At this wavelength the complex dielectric constant of silver is $\varepsilon = -1.42 + 0.32i$. Plot the dipole strength $|\mathbf{p}|^2$ as a function of the normalized height z_o/a in the range $[1\,..\,10]$.
- Calculate the scattered radiation pattern in the upper half space.