OPT 462: Homework Set 5   Due on February 21, 2001

1   Free space dyadic Green's function

- Use the free space scalar Green's function G_o to derive the dyadic Green's function $\overset{\leftrightarrow}{\mathbf{G}}$ for time-harmonic fields.

- Split $\overset{\leftrightarrow}{\mathbf{G}}$ into a near-field term $\overset{\leftrightarrow}{\mathbf{G}}_{nf}$, an intermediate-field term $\overset{\leftrightarrow}{\mathbf{G}}_{if}$, and a farfield term $\overset{\leftrightarrow}{\mathbf{G}}_{ff}$. $\overset{\leftrightarrow}{\mathbf{G}}_{nf}$ decays as $|\mathbf{r}-\mathbf{r}'|^{-3}$, $\overset{\leftrightarrow}{\mathbf{G}}_{if}$ as $|\mathbf{r}-\mathbf{r}'|^{-2}$, and $\overset{\leftrightarrow}{\mathbf{G}}_{ff}$ as $|\mathbf{r}-\mathbf{r}'|^{-1}$.

2   Dipole-dipole interactions

The current density $\mathbf{j}(\mathbf{r},t)$ of a particle (atom, molecule, quantum dot, ...) defined by a neutral collection of charges can be expanded in a Taylor series with respect to the center of mass coordinate $\mathbf{r}_n$. In the dipole approximation only the lowest order term is retained and $\mathbf{j}(\mathbf{r},t)$ reads as

$$\mathbf{j}(\mathbf{r},t) \;=\; \frac{d\mathbf{p}_n(t)}{dt}\,\delta(\mathbf{r}-\mathbf{r}_n)\,, \tag{1}$$

where $\mathbf{p}_n(t)$ is the dipole moment defined by the particular charge distribution.

- Assume there are N different dipolar particles with dipole moments $\mathbf{p}_1, .., \mathbf{p}_N$ oscillating coherently at a single frequency ω. The center of mass coordinates are $\mathbf{r}_1, .., \mathbf{r}_N$. Determine in general form the electric field $\mathbf{E}(\mathbf{r},t)$ generated by these dipoles.

- In the absence of any exterior driving field $\mathbf{E}_o(\mathbf{r},t)$ the dipole moments are zero (no permanent dipole moments). The dipole moment $\mathbf{p}_n$ is induced by the local field $\mathbf{E}_l(\mathbf{r}_n,t)$ due to the driving field $\mathbf{E}_o$ and the partial fields of the other nanoparticles. In the linear regime the following relationship holds between the complex amplitudes

$$\mathbf{p}_n(\omega) \;=\; \alpha_n(\omega)\,\mathbf{E}_l(\mathbf{r}_n,\omega)\,. \tag{2}$$

Here, α is the linear, isotropic and spatially non-dispersive polarizability. The particular form of α depends on the physical properties of the nanoparticle. Write down in general form a system of N equations to determine the dipole moments $\mathbf{p}_1, .., \mathbf{p}_N$.

- Consider two particles in free space as shown in the figure. One of the particles acts as a local probe whereas the other particle represents a sample particle. The probing particle is locally excited by a monochromatic plane wave ($\lambda = 800\,nm$) incident along the x-direction and polarized along the z-direction. The amplitude of the plane wave is E_o. The particles have identical radii of $a = 5\,nm$. The optical properties of both particles are determined by their dielectric constants $\varepsilon = -20 + 0.8i$ (probe) and $\varepsilon = 2.5$ (sample). In the nonretarded limit, the polarizabilities are calculated as

$$\alpha = 4\pi\varepsilon_o\,a^3\,\frac{\varepsilon - 1}{\varepsilon + 2}\,. \tag{3}$$

The sample particle is scanned along the x-direction. Plot the dipole strength of both particles as a function of x for the three different probe heights $z_o = 20\,nm,\ 50\,nm,\ 100\,nm$. Use the plot range $x/\lambda = [-0.5, .., 0.5]$. Compare your result with a nonretarded calculation for which only $\overset{\leftrightarrow}{\mathbf{G}}_{nf}$ is included in the Green's function.