OPT 462: Homework Set 4 Due on February 14, 2001

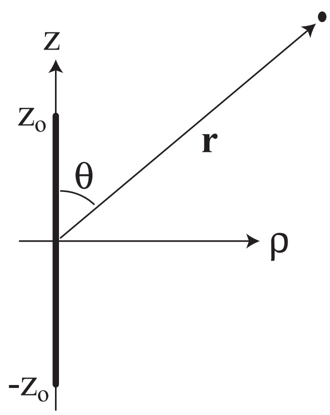
1 Gauge Transformations

The choice of $\nabla \cdot \mathbf{A}(\mathbf{r}, t)$ defines a particular gauge. With the gauge transformations we can move from one gauge to another.

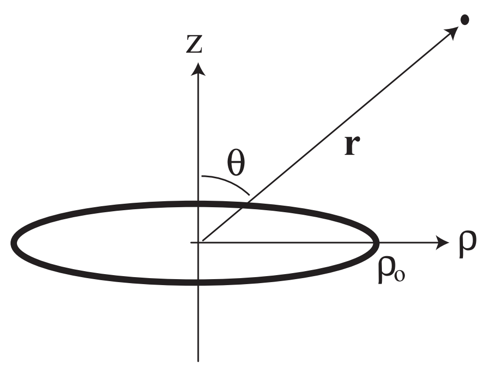
- Find the gauge transformation which transforms the potentials $\mathbf{A}$, ϕ from the Lorentz gauge to the Coulomb gauge.

2 Line Sources and Ring Sources

Consider the time-harmonic current densities shown in the figure below. Although we cannot analytically solve for the fields we can exactly derive the farfields and the radiated power.



$$\mathbf{j}(\mathbf{r}) = I_o \cos[\frac{(2n+1)\pi}{2 z_o} z]\, \delta(x)\delta(y)\mathbf{n}_z$$



$$\mathbf{j}(\mathbf{r}) = I_o \cos(n\,\phi)\; \delta(\rho - \rho_o)\delta(z)\mathbf{n}_\phi$$

- Determine the charge densities.
- Determine expressions for the vector and scalar potentials in the Lorentz gauge and simplify as much as possible.
- At sufficiently large distances we can use the approximation

$$|\mathbf{r} - \mathbf{r}'| \; = \; \sqrt{r^2 + r'^2 - 2(\mathbf{r}\cdot\mathbf{r}')} \; \approx \; r - \mathbf{r}\cdot\mathbf{r}'/r \; . \tag{1}$$

  Show that this approximation follows from a series expansion. The scalar Green's function can now be written as

$$G_o(\mathbf{r}, \mathbf{r}') \; = \; \frac{\exp(ik[r - \mathbf{r}\cdot\mathbf{r}'/r])}{4\pi r} \; . \tag{2}$$

  Use this expression to calculate the vector and scalar potentials in the farfield ($r \gg r'$).

- Derive the corresponding electric and magnetic fields.

- Determine the total radiated power P for each of the two current distributions. Does it depend on the integer n? Why?

- Choose $n=0$ for the line current and $n=1$ for the ring current. Plot the radiation patterns

$$\frac{p(\theta,\phi)}{P}\,, \tag{3}$$

where $p(\theta,\phi)$ is the energy flux per unit solid angle, i.e.

$$P = \int_0^{\pi}\int_0^{2\pi} p(\theta,\phi)\,\sin\theta\; d\theta d\phi\,. \tag{4}$$