OPT 462: Homework Set 3 — Due on February 7, 2001

1 Photonic Crystals

A photonic crystal is an inhomogeneous medium with periodic dielectric and magnetic properties. The periodicity of this structure gives rise to forbidden frequency bands in which the light can not propagate through the medium. Photonic crystals have very promising potential applications as waveguides, lasers, mirrors, etc. For a general overview on this topic you can take a look at the paper: J. D. Joannopoulos, P. R. Villenueve, and S. Fan, *Nature* **386**, 143 (1997).

Let us consider a non-magnetic ($\mu = 1$) photonic crystal with a periodic dielectric constant $\varepsilon(x, y, z)$ in one-dimension

$$\varepsilon(x, y, z+d) = \varepsilon(x, y, z), \tag{1}$$

where d is the period of the crystal. The inhomogeneous dielectric constant can be written as

$$\varepsilon(x, y, z) = \begin{cases} \varepsilon_1 & 0 < z < l \\ \varepsilon_2 & l < z < d \end{cases} . \tag{2}$$

- Assume that you have two plane waves in each layer, one is propagating in the positive z direction and the other in the negative z direction. Find first an expression that relates the plane wave amplitudes of the n^{th} to the plane wave amplitudes of the $(n+1)^{th}$ layer of the same material. Then use the Flocquet (Bloch) theorem[1] which states: If ϕ is a field in a periodic medium then the solution for ϕ can be written in the following form

  $$\phi(z+d) = e^{ik_{Bz}d}\phi(z), \tag{3}$$

  where k_{Bz} is the Bloch wavevector. Determine the dispersion relation [eigenvalue equation relating ω and $\mathbf{k} = (k_x, k_y, k_{Bz})$], separately for TE and TM modes.

- Plot, separately for TE and TM modes, the band diagramm $\omega(k_{||})$, with $k_{||}$ being the transverse propagation constant $k_{||} = \sqrt{k_x^2 + k_y^2}$. Highlight the allowed bands. Use the normalized coordinates $\omega\, d/c$ and $k_{||}\, d$. Use the following values: $\varepsilon_1 = 17.88$ (InSb), $\varepsilon_2 = 2.31$ (quartz), $l/d = 1/3$ and $(d-l)/d = 2/3$. When you plot the band diagram be aware that you might have propagating modes even if $\sqrt{\omega^2 \varepsilon_{1,2}/c^2 - k_{||}^2}$ is imaginary.

- The Brewster effect is a distinctive feature of TM modes. How is this effect manifested in the plot that you obtained? Explain what is happening physically. Find a mathematical expression that describes the effect on the plot.

- Lately, several articles have been written on building an *omnidirectional* mirror. How can the photonic crystal be used as a mirror? Is it true that it is *omnidirectional*? Why?

[1] Flocquet formulated the theorem almost a century before Bloch. However, not in the context of quantum mechanics.