OPT 462: Final Exam May 12, 2001

(Max. points = 120. No more than 100 points required for grade **A**)

1 Harmonic Oscillator

- (10 points) Determine the total energy radiated by an *undriven* harmonic oscillator.
- (5 points) Is it better to have a high Q or a low Q oscillator ($Q = \gamma/\omega_o$) for maximizing the radiated energy ?

2 Longitudinal Field of a Gaussian Beam

A monochromatic laser beam is characterized by its two-component electric field amplitude $\mathbf{E}(\mathbf{r}) = [E_x(\mathbf{r}), 0, E_z(\mathbf{r})]$. In the plane $z = 0$ the x-component has a Gaussian beam profile given by

$$E_x(x, y, z = 0) \;=\; E_o \, \mathrm{e}^{-(x^2+y^2)/w_o^2} \; . \tag{1}$$

- (5 points) Derive the field E_x for any z (do not solve any integrals).
- (10 points) Determine the field E_z (do not solve any integrals).
- (10 points) Calculate the farfield and show that it is a spherical wave.

3 Propagation of a Lorentzian Light Pulse

At the time $t = 0$ the spatial profile of a laser pulse is characterized by the field

$$\mathbf{E}(\mathbf{r}) \;=\; E_o \frac{1}{1 + (z/z_o)^2} \cos\!\left(\frac{\omega_o}{c} z\right) \mathbf{n}_x \; . \tag{2}$$

- (5 points) Determine the field for any time t.
- (10 points) Calculate and draw the frequency spectrum $\hat{\mathbf{E}}(\mathbf{r}, \omega)$.
- (5 points) Determine the energy per unit area and unit angular frequency $dW/(dA\, d\omega)$.
- (10 points) Determine the energy per unit area dW/dA.
- (5 points) Assume that the pulse is propagating in a dispersive medium characterized by the index of refraction $n(\omega)$ and determine the corresponding electric field $\mathbf{E}(\mathbf{r}, t)$.

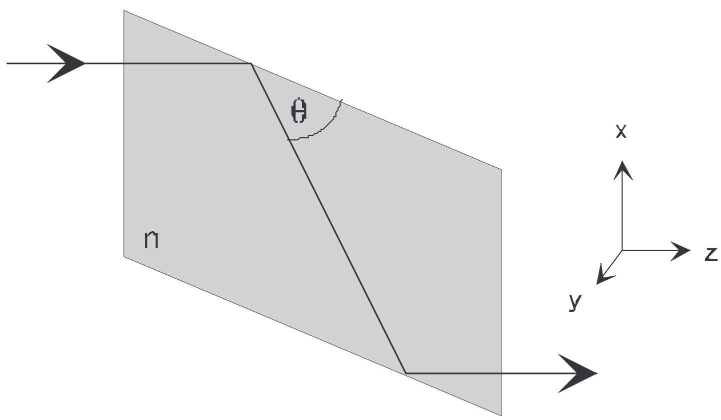
Hint:
$$\int_{-\infty}^{\infty} \frac{\cos(x)}{(1 + (x/x_o)^2)} \exp[i\alpha x] dx \;=\; \frac{\pi\, x_o}{2} \left[\exp(-x_o|\alpha - 1|) \,+\, \exp(-x_o|\alpha + 1|)\right] \; . \tag{3}$$

4 Light Scattering by an Elliptical Particle

An elliptical particle with its long axis along the x-axis is irradiated by a monochromatic plane wave propagating along the z-axis. The electric field amplidude of the plane wave is E_o and its polarization is along the x-axis. For simplicity the particle is modeled by two separate spherical particles with polarizabilities α which are located at $(0, 0, -x_o)$ and $(0, 0, x_o)$. Assume that the two particles interact only through their near-fields and ignore any retardation effects.

- (15 points) Determine the dipole moment in each of the two spherical particles.
- (10 points) The dipole moment $\mathbf{p}_{ell}$ of the elliptical particle can be assumed to be the sum of the dipole moments of the two spherical particles. Determine the effective polarizability α_{eff} defined by $\mathbf{p}_{ell} = \alpha_{eff}\, \mathbf{E}_o$.

5 The Fresnel Rhomb



A linearly polarized monochromatic plane wave propagates in z-direction and hits the endface of a rhomb at normal incidence (see figure). The plane wave is polarized in the (x, y)-plane at 45°. Inside the rhomb the plane wave undergoes two successive total internal reflections. Each total internal reflection delays one polarization component versus the other by a phase which depends on the angle θ. The index of refraction of the rhomb is n and the medium surrounding the rhomb is air.

- (20 points) Determine the angle θ for which the incoming plane wave is converted into a circularly polarized plane wave.