# Appendix D

# Farfield Green's Functions

In this Appendix we state the asymptotic farfield Green's functions for a planarly layered medium. It is assumed that the source point $\mathbf{r}_o = (x_o, y_o, z_o)$ is in the upper half-space ($z > 0$). The field is evaluated in a point $\mathbf{r} = (x, y, z)$ in the farzone, i.e. $r \gg \lambda$. The optical properties of the upper half-space and the lower half-space are characterized by $\varepsilon_1$, $\mu_1$ and $\varepsilon_n$, $\mu_n$, respectively. The planarly layered medium in between of the two half-spaces is characterized by the generalized Fresnel reflection and transmission coefficients. We choose a coordinate system with origin on the topmost surface of the layered medium with the $z$-axis perpendicular to the interfaces. In this case, $z_o$ denotes the height of the point source relative to the topmost layer. In the upper half-space, the asymptotic dyadic Green's function is defined as

$$\mathbf{E}(\mathbf{r}) \;=\; \frac{\omega^2}{\varepsilon_o\, c^2}\mu_1 \left[\overleftrightarrow{\mathbf{G}}_o(\mathbf{r},\mathbf{r}_o) + \overleftrightarrow{\mathbf{G}}_{ref}(\mathbf{r},\mathbf{r}_o)\right] \boldsymbol{\mu}\,, \tag{D.1}$$

where $\boldsymbol{\mu}$ is the dipole moment of a dipole located at $\mathbf{r}_o$ and $\overleftrightarrow{\mathbf{G}}_o$ and $\overleftrightarrow{\mathbf{G}}_{ref}$ are the primary and reflected parts of the Green's function. In the lower half-space we define

$$\mathbf{E}(\mathbf{r}) \;=\; \frac{\omega^2}{\varepsilon_o\, c^2}\mu_1\, \overleftrightarrow{\mathbf{G}}_{tr}(\mathbf{r},\mathbf{r}_o)\, \boldsymbol{\mu}\,, \tag{D.2}$$

$\overleftrightarrow{\mathbf{G}}_{tr}$ being the transmitted part of the Green's function. The asymptotic Green's functions can be derived by using the farfield forms of the angular spectrum representation.

The primary Green's function in the farzone is derived as

$$\overleftrightarrow{\mathbf{G}}_o(\mathbf{r},\mathbf{r}_o) \;=\; \frac{\exp(ik_1 r)}{4\pi r}\, \exp\!\big[-ik_1(x_o\, x/r + y_o\, y/r + z_o\, z/r)\big] \times \tag{D.3}$$

$$\begin{bmatrix} (1-x^2/r^2) & -xy/r^2 & -xz/r^2 \\ -xy/r^2 & (1-y^2/r^2) & -yz/r^2 \\ -xz/r^2 & -yz/r^2 & (1-z^2/r^2) \end{bmatrix}.$$

The reflected part of the Green's function in the farzone is

$$\overleftrightarrow{\mathbf{G}}_{ref}(\mathbf{r},\mathbf{r}_o) = \frac{\exp(ik_1 r)}{4\pi r}\,\exp\big[-ik_1(x_o\,x/r + y_o\,y/r - z_o\,z/r)\big]\times \tag{D.4}$$
$$\begin{bmatrix} x^2\!/\rho^2\, z^2\!/r^2\,\Phi_1^{(2)} + y^2\!/\rho^2\,\Phi_1^{(3)} & xy/\rho^2\, z^2\!/r^2\,\Phi_1^{(2)} - xy/\rho^2\,\Phi_1^{(3)} & -xz/r^2\,\Phi_1^{(1)} \\ xy/\rho^2\, z^2\!/r^2\,\Phi^{(2)} - xy/\rho^2\,\Phi_1^{(3)} & y^2\!/\rho^2\, z^2\!/r^2\,\Phi^{(2)} + x^2\!/\rho^2\,\Phi_1^{(3)} & -yz/r^2\,\Phi_1^{(1)} \\ -xz/r^2\,\Phi_1^{(2)} & -yz/r^2\,\Phi_1^{(2)} & (1-z^2\!/r^2)\,\Phi_1^{(1)} \end{bmatrix},$$

where the potentials are determined in terms of the generalized reflection coefficients of the layered structure as

$$\left.\begin{aligned} \Phi_1^{(1)} &= r^p(k_\rho) \\ \Phi_1^{(2)} &= -r^p(k_\rho) \\ \Phi_1^{(3)} &= r^s(k_\rho) \end{aligned}\right\}\quad k_\rho = k_1\,\rho/r\ . \tag{D.5}$$

The transmitted part of the Green's function in the farzone is

$$\overleftrightarrow{\mathbf{G}}_{tr}(\mathbf{r},\mathbf{r}_o) = \frac{\exp[ik_n(r+\delta\,z/r)]}{4\pi r}\,\exp\!\left[-ik_1\!\left(x_o\,x/r + y_o\,y/r - z_o\sqrt{1-n_n^2/n_1^2\,\rho^2\!/r^2}\right)\right]\times$$
$$\begin{bmatrix} x^2\!/\rho^2\, z^2\!/r^2\,\Phi_n^{(2)} + y^2\!/\rho^2\,\Phi_n^{(3)} & xy/\rho^2\, z^2\!/r^2\,\Phi_n^{(2)} - xy/\rho^2\,\Phi_n^{(3)} & -xz/r^2\,\Phi_n^{(1)} \\ xy/\rho^2\, z^2\!/r^2\,\Phi_n^{(2)} - xy/\rho^2\,\Phi_n^{(3)} & y^2\!/\rho^2\, z^2\!/r^2\,\Phi_n^{(2)} + x^2\!/\rho^2\,\Phi_n^{(3)} & -yz/r^2\,\Phi_n^{(1)} \\ -xz/r^2\,\Phi_n^{(2)} & -yz/r^2\,\Phi_n^{(2)} & (1-z^2\!/r^2)\,\Phi_n^{(1)} \end{bmatrix}. \tag{D.6}$$

Here, the potentials are determined in terms of the generalized transmission coefficients of the layered structure as

$$\left.\begin{aligned} \Phi_n^{(1)} &= t^p(k_\rho)\,\frac{n_n}{n_1}\,\frac{k_n\,z/r}{\sqrt{k_1^2-k_\rho^2}} \\ \Phi_n^{(2)} &= -t^p(k_\rho)\,\frac{n_n}{n_1} \\ \Phi_n^{(3)} &= t^s(k_\rho)\,\frac{k_n\,z/r}{\sqrt{k_1^2-k_\rho^2}} \end{aligned}\right\}\quad k_\rho = k_n\,\rho/r\ , \tag{D.7}$$

where $\delta$ denotes the overall thickness of the layered structure. A vertical dipole is described by the potential $\Phi^{(1)}$ alone and gives rise to purely $p$ polarized fields. On the other hand, a horizontal dipole is represented by $\Phi^{(2)}$ and $\Phi^{(3)}$ and its field contains both $s$ and $p$ polarized components. The coordinates $(x,y,z)$ can be substituted by the spherical angles $\theta$ and $\phi$. For angles $\alpha = \pi - \theta$ beyond the critical angle of $\alpha_c = \arcsin(n_1/n_n)$ the field depends exponentially on the height $z_o$.