Appendix C

Fields of a Dipole Near a Layered Substrate

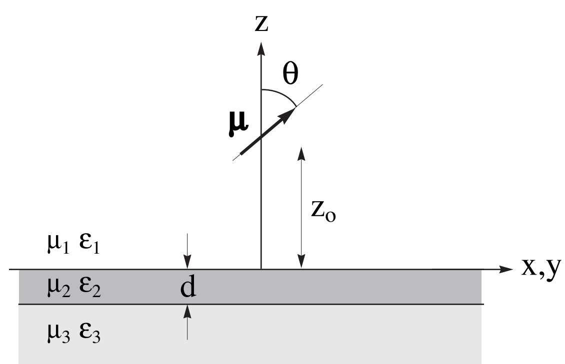

Figure C.1: An electric dipole with moment $\boldsymbol{\mu}$ is located at $\mathbf{r}_o = (0,0,z_o)$ near a layered substrate. The fields in each medium are expressed in cylindrical coordinates $\mathbf{r} = (\rho, \varphi, z)$.

Vertical Electric Dipole

The cylindrical field components of a vertically oriented dipole $\boldsymbol{\mu} = (0,0,\mu_z)$ read as

$$E_{1\rho} \;=\; \rho\,(z-z_o)\,\frac{\mu_z}{4\pi\,\varepsilon_o\,\varepsilon_1}\,\frac{e^{ik_1 R_o}}{R_o^3}\,\Big[\frac{3}{R_o^2}-\frac{3\,ik_1}{R_o}-k_1^2\Big] - \frac{i\,\mu_z}{4\pi\,\varepsilon_o\,\varepsilon_1}\int\limits_0^\infty dk_\rho\,J_1(k_\rho\,\rho)\,A_1\,k_\rho\,k_{1z}\,e^{ik_{1z}(z+z_o)} \tag{C.1}$$

$$E_{2\rho} = \frac{i\,\mu_z}{4\pi\,\varepsilon_o\,\varepsilon_1} \int\limits_0^\infty dk_\rho\, J_1(k_\rho\,\rho)\left[A_2\, e^{-ik_{2z}z} - A_3\, e^{ik_{2z}z}\right] k_\rho\, k_{2z}\, e^{ik_{1z}z_o} \tag{C.2}$$

$$E_{3\rho} = \frac{i\,\mu_z}{4\pi\,\varepsilon_o\,\varepsilon_1} \int\limits_0^\infty dk_\rho\, J_1(k_\rho\,\rho)\, A_4\, k_\rho\, k_{3z}\, e^{i(k_{1z}z_o - k_{3z}z)} \tag{C.3}$$

$$E_{1\varphi} = E_{2\varphi} = E_{3\varphi} = 0 \tag{C.4}$$

$$\begin{aligned} E_{1z} = {} & \frac{\mu_z}{4\pi\,\varepsilon_o\,\varepsilon_1} \frac{e^{ik_1R_o}}{R_o}\left[\frac{3\,(z-z_o)^2}{R_o^4} - \frac{3ik_1\,(z-z_o)^2}{R_o^3} - \frac{1+k_1^2\,(z-z_o)^2}{R_o^2} + \frac{ik_1}{R_o} + k_1^2\right] \\ & + \frac{\mu_z}{4\pi\,\varepsilon_o\,\varepsilon_1} \int\limits_0^\infty dk_\rho\, J_0(k_\rho\,\rho)\, A_1\, k_\rho^2\, e^{ik_{1z}(z+z_o)} \end{aligned} \tag{C.5}$$

$$E_{2z} = \frac{\mu_z}{4\pi\,\varepsilon_o\,\varepsilon_1} \int\limits_0^\infty dk_\rho\, J_0(k_\rho\,\rho)\left[A_2\, e^{-ik_{2z}z} + A_3\, e^{ik_{2z}z}\right] k_\rho^2\, e^{ik_{1z}z_o} \tag{C.6}$$

$$E_{3z} = \frac{\mu_z}{4\pi\,\varepsilon_o\,\varepsilon_1} \int\limits_0^\infty dk_\rho\, J_0(k_\rho\,\rho)\, A_4\, k_\rho^2\, e^{i(k_{1z}z_o - k_{3z}z)} \tag{C.7}$$

$$H_{1\rho} = H_{2\rho} = H_{3\rho} = 0 \tag{C.8}$$

$$\begin{aligned} H_{1\varphi} = {} & -\frac{i\,\omega\,\mu_z}{4\pi}\,\rho\,\frac{e^{ik_1R_o}}{R_o^2}\left[\frac{1}{R_o} - ik_1\right] \\ & -\frac{i\,\omega\,\mu_z}{4\pi} \int\limits_0^\infty dk_\rho\, J_1(k_\rho\,\rho)\, A_1\, k_\rho\, e^{ik_{1z}(z+z_o)} \end{aligned} \tag{C.9}$$

$$H_{2\varphi} = -\frac{i\,\omega\,\varepsilon_2\,\mu_z}{4\pi\,\varepsilon_1} \int\limits_0^\infty dk_\rho\, J_1(k_\rho\,\rho)\left[A_2\, e^{-ik_{2z}z} + A_3\, e^{ik_{2z}z}\right] k_\rho\, e^{ik_{1z}z_o} \tag{C.10}$$

$$H_{3\varphi} = -\frac{i\,\omega\,\varepsilon_3\,\mu_z}{4\pi\,\varepsilon_1} \int\limits_0^\infty dk_\rho\, J_1(k_\rho\,\rho)\, A_4\, k_\rho\, e^{i(k_{1z}z_o - k_{3z}z)} \tag{C.11}$$

$$H_{1z} = H_{2z} = H_{3z} = 0 \tag{C.12}$$

Horizontal Electric Dipole

The cylindrical field components of a horizontally oriented dipole $\boldsymbol{\mu}=(\mu_x,0,0)$ read as

$$
\begin{aligned}
E_{1\rho} \;=\; & \cos\varphi\,\frac{\mu_x}{4\pi\,\varepsilon_o\,\varepsilon_1}\,\frac{e^{ik_1R_o}}{R_o}\Big[\big[\,k_1^2+\frac{ik_1}{R_o}-\frac{1}{R_o^2}\,\big]\;+\;\frac{\rho^2}{R_o^3}\,\big[\,\frac{3}{R_o^2}-\frac{3\,ik_1}{R_o}-k_1^2\,\big]\Big]\;+\\
& \cos\varphi\,\frac{\mu_x}{4\pi\,\varepsilon_o\,\varepsilon_1}\int_0^\infty dk_\rho\;e^{ik_{1z}\,(z+z_o)}\,\Big\{\frac{1}{\rho}\,J_1(k_\rho\,\rho)\,\big[k_\rho B_1-ik_{1z}C_1\big]\\
& \qquad -\,i\,k_{1z}\,J_0(k_\rho\,\rho)\,\big[ik_{1z}B_1-k_\rho C_1\big]\Big\}
\end{aligned}
\tag{C.13}
$$

$$
\begin{aligned}
E_{2\rho} \;=\; & \cos\varphi\,\frac{\mu_x}{4\pi\,\varepsilon_o\,\varepsilon_1}\int_0^\infty dk_\rho\;e^{ik_{1z}\,z_o}\,\Big\{\frac{1}{\rho}\,J_1(k_\rho\,\rho)\,\Big[\big[k_\rho B_2+ik_{2z}C_2\big]\,e^{-ik_{2z}z}\\
& \qquad +\big[k_\rho B_3-ik_{2z}C_3\big]\,e^{ik_{2z}z}\Big]\\
& \qquad -\,i\,k_{2z}\,J_0(k_\rho\,\rho)\,\Big[\big[ik_{2z}B_2+k_\rho C_2\big]\,e^{-ik_{2z}z}\\
& \qquad +\big[ik_{2z}B_3-k_\rho C_3\big]\,e^{ik_{2z}z}\Big]\Big\}
\end{aligned}
\tag{C.14}
$$

$$
\begin{aligned}
E_{3\rho} \;=\; & \cos\varphi\,\frac{\mu_x}{4\pi\,\varepsilon_o\,\varepsilon_1}\int_0^\infty dk_\rho\;e^{i(k_{1z}\,z_o-k_{3z}\,z)}\,\Big\{\frac{1}{\rho}\,J_1(k_\rho\,\rho)\,\big[k_\rho B_4+ik_{3z}C_4\big]\\
& \qquad -\,i\,k_{3z}\,J_0(k_\rho\,\rho)\,\big[ik_{3z}B_4+k_\rho C_4\big]\Big\}
\end{aligned}
\tag{C.15}
$$

$$
\begin{aligned}
E_{1\varphi} \;=\; & \sin\varphi\,\frac{\mu_x}{4\pi\,\varepsilon_o\,\varepsilon_1}\,\frac{e^{ik_1R_o}}{R_o}\,\big[\,\frac{1}{R_o^2}-\frac{ik_1}{R_o}-k_1^2\,\big]\;+\\
& \sin\varphi\,\frac{\mu_x}{4\pi\,\varepsilon_o\,\varepsilon_1}\int_0^\infty dk_\rho\;e^{ik_{1z}\,(z+z_o)}\,\Big\{\frac{1}{\rho}\,J_1(k_\rho\,\rho)\,\big[k_\rho B_1-ik_{1z}C_1\big]\\
& \qquad -\,k_1^2\,J_0(k_\rho\,\rho)\,B_1\big]\Big\}
\end{aligned}
\tag{C.16}
$$

$$
\begin{aligned}
E_{2\varphi} \;=\; & \sin\varphi\,\frac{\mu_x}{4\pi\,\varepsilon_o\,\varepsilon_1}\int_0^\infty dk_\rho\;e^{ik_{1z}\,z_o}\,\Big\{\frac{1}{\rho}\,J_1(k_\rho\,\rho)\,\Big[\big[k_\rho B_2+ik_{2z}C_2\big]\,e^{-ik_{2z}z}\\
& \qquad +\big[k_\rho B_3-ik_{2z}C_3\big]\,e^{ik_{2z}z}\Big]\\
& \qquad -\,k_2^2\,J_0(k_\rho\,\rho)\,\big[B_2+B_3\,\big]\Big\}
\end{aligned}
\tag{C.17}
$$

$$E_{3\varphi} = \sin\varphi \frac{\mu_x}{4\pi\varepsilon_o\varepsilon_1} \int_0^\infty dk_\rho\, e^{i(k_{1z}\,z_o - k_{3z}\,z)} \Big\{ \frac{1}{\rho} J_1(k_\rho\,\rho) \big[k_\rho B_4 + ik_{3z}C_4\big] - k_3^2\, J_0(k_\rho\,\rho)\, B_4 \,\big]\Big\} \tag{C.18}$$

$$E_{1z} = \cos\varphi \frac{\mu_x}{4\pi\varepsilon_o\varepsilon_1}\, \rho\,(z-z_o)\, \frac{e^{ik_1R_o}}{R_o^3} \Big[\frac{3}{R_o^2} - \frac{3ik_1}{R_o} - k_1^2 \Big] - \cos\varphi \frac{\mu_x}{4\pi\varepsilon_o\varepsilon_1} \int_0^\infty dk_\rho\, e^{ik_{1z}\,(z+z_o)}\, k_\rho\, J_1(k_\rho\,\rho) \big[ik_{1z}B_1 - k_\rho C_1\big] \tag{C.19}$$

$$E_{2z} = \cos\varphi \frac{\mu_x}{4\pi\varepsilon_o\varepsilon_1} \int_0^\infty dk_\rho\, e^{ik_{1z}\,z_o} \Big\{ k_\rho\, J_1(k_\rho\,\rho) \Big[\big[ik_{2z}B_2 + k_\rho C_2\big]\, e^{-ik_{2z}z} - \big[ik_{2z}B_3 - k_\rho C_3\big]\, e^{ik_{2z}z} \Big]\Big\} \tag{C.20}$$

$$E_{3z} = \cos\varphi \frac{\mu_x}{4\pi\varepsilon_o\varepsilon_1} \int_0^\infty dk_\rho\, e^{i(k_{1z}\,z_o - k_{3z}\,z)}\, k_\rho\, J_1(k_\rho\,\rho) \big[ik_{3z}B_4 + k_\rho C_4\big] \tag{C.21}$$

$$H_{1\rho} = \sin\varphi \frac{i\,\omega\,\mu_x}{4\,\pi}\, (z-z_o)\, \frac{e^{ik_1R_o}}{R_o^2} \Big[\frac{1}{R_o} - ik_1 \Big] + \sin\varphi \frac{i\,\omega\,\mu_x}{4\,\pi} \int_0^\infty dk_\rho\, e^{ik_{1z}\,(z+z_o)} \Big\{ \frac{1}{\rho} J_1(k_\rho\,\rho)\, C_1 \;-\; ik_{1z}\, J_0(k_\rho\,\rho)\, B_1 \Big\} \tag{C.22}$$

$$H_{2\rho} = \sin\varphi \frac{i\,\omega\,\varepsilon_2\,\mu_x}{4\,\pi\,\varepsilon_1} \int_0^\infty dk_\rho\, e^{ik_{1z}\,z_o} \Big\{ \frac{1}{\rho} J_1(k_\rho\,\rho) \big[C_2\, e^{-ik_{2z}z} + C_3\, e^{ik_{2z}z}\big] - ik_{2z}\, J_0(k_\rho\,\rho) \big[B_2\, e^{-ik_{2z}z} - B_3\, e^{ik_{2z}z} \,\big]\Big\} \tag{C.23}$$

$$H_{3\rho} = \sin\varphi \frac{i\,\omega\,\varepsilon_3\,\mu_x}{4\,\pi\,\varepsilon_1} \int_0^\infty dk_\rho\, e^{i(k_{1z}\,z_o - k_{3z}\,z)} \Big\{ \frac{1}{\rho} J_1(k_\rho\,\rho)\, C_4 + ik_{3z}\, J_0(k_\rho\,\rho)\, B_4 \Big\} \tag{C.24}$$

$$H_{1\varphi} = \cos\varphi \frac{i\,\omega\,\mu_x}{4\,\pi}\, (z-z_o)\, \frac{e^{ik_1R_o}}{R_o^2} \Big[\frac{1}{R_o} - ik_1 \Big] - \tag{C.25}$$

$$\cos\varphi\,\frac{i\,\omega\,\mu_x}{4\,\pi}\int_0^\infty dk_\rho\, e^{ik_{1z}\,(z+z_o)}\,\Big\{\frac{1}{\rho}\,J_1(k_\rho\,\rho)\,C_1 + J_0(k_\rho\,\rho)\,\big[ik_{1z}B_1 - k_\rho C_1\big]\Big\}$$

$$H_{2\varphi} \;=\; \cos\varphi\,\frac{i\,\omega\,\varepsilon_2\,\mu_x}{4\,\pi\,\varepsilon_1}\int_0^\infty dk_\rho\, e^{ik_{1z}\,z_o}\,\Big\{\frac{1}{\rho}\,J_1(k_\rho\,\rho)\,\big[C_2\,e^{-ik_{2z}z} + C_3\,e^{ik_{2z}z}\big] - J_0(k_\rho\,\rho)\,\Big[\big[ik_{2z}B_2 + k_\rho C_2\big]\,e^{-ik_{2z}z} - \big[ik_{2z}B_3 - k_\rho C_3\big]\,e^{ik_{2z}z}\Big]\Big\} \qquad \text{(C.26)}$$

$$H_{3\varphi} \;=\; \cos\varphi\,\frac{i\,\omega\,\varepsilon_3\,\mu_x}{4\,\pi\,\varepsilon_1}\int_0^\infty dk_\rho\, e^{i(k_{1z}\,z_o - k_{3z}\,z)}\,\Big\{\frac{1}{\rho}\,J_1(k_\rho\,\rho)\,C_4 - J_0(k_\rho\,\rho)\,\big[ik_{3z}B_4 + k_\rho C_4\big]\Big\} \qquad \text{(C.27)}$$

$$H_{1z} \;=\; -\sin\varphi\,\frac{i\,\omega\,\mu_x}{4\,\pi}\,\rho\,\frac{e^{ik_1 R_o}}{R_o^2}\,\Big[\,\frac{1}{R_o} - ik_1\,\Big] \;-\; \sin\varphi\,\frac{i\,\omega\,\mu_x}{4\,\pi}\int_0^\infty dk_\rho\, e^{ik_{1z}\,(z+z_o)}\,k_\rho\,J_1(k_\rho\,\rho)\,B_1 \qquad \text{(C.28)}$$

$$H_{2z} \;=\; -\sin\varphi\,\frac{i\,\omega\,\varepsilon_2\,\mu_x}{4\,\pi\,\varepsilon_1}\int_0^\infty dk_\rho\, e^{ik_{1z}\,z_o}\,k_\rho\,J_1(k_\rho\,\rho)\,\big[B_2\,e^{-ik_{2z}z} + B_3\,e^{ik_{2z}z}\big] \qquad \text{(C.29)}$$

$$H_{3z} \;=\; -\sin\varphi\,\frac{i\,\omega\,\varepsilon_3\,\mu_x}{4\,\pi\,\varepsilon_1}\int_0^\infty dk_\rho\, e^{i(k_{1z}\,z_o - k_{3z}\,z)}\,k_\rho\,J_1(k_\rho\,\rho)\,B_4 \qquad \text{(C.30)}$$

Definition of the coefficients $\mathrm{A_j}$, $\mathrm{B_j}$, and $\mathrm{C_j}$

The coefficients A_j, B_j, C_j are determined by the boundary conditions on the interfaces. Using the abbreviations

$$\begin{aligned} f_1 &= \varepsilon_2\,k_{1z} - \varepsilon_1\,k_{2z} & g_1 &= \mu_2\,k_{1z} - \mu_1\,k_{2z} \\ f_2 &= \varepsilon_2\,k_{1z} + \varepsilon_1\,k_{2z} & g_2 &= \mu_2\,k_{1z} + \mu_1\,k_{2z} \\ f_3 &= \varepsilon_3\,k_{2z} - \varepsilon_2\,k_{3z} & g_3 &= \mu_3\,k_{2z} - \mu_2\,k_{3z} \\ f_4 &= \varepsilon_3\,k_{2z} + \varepsilon_2\,k_{3z} & g_4 &= \mu_3\,k_{2z} + \mu_2\,k_{3z}\quad, \end{aligned} \qquad \text{(C.31)}$$

the coefficients read as:

$$A_1(k_\rho) = i\,\frac{k_\rho\,(f_1 f_4 + f_2 f_3\,e^{2ik_{2z}d})}{k_{1z}\,(f_2 f_4 + f_1 f_3\,e^{2ik_{2z}d})} \tag{C.32}$$

$$A_2(k_\rho) = i\,\frac{2\,\varepsilon_1\,k_\rho\,f_4}{(f_2 f_4 + f_1 f_3\,e^{2ik_{2z}d})} \tag{C.33}$$

$$A_3(k_\rho) = i\,\frac{2\,\varepsilon_1\,k_\rho\,f_3\,e^{2ik_{2z}d}}{(f_2 f_4 + f_1 f_3\,e^{2ik_{2z}d})} \tag{C.34}$$

$$A_4(k_\rho) = i\,\frac{4\,\varepsilon_1\varepsilon_2\,k_\rho\,k_{2z}\,e^{i(k_{2z}-k_{3z})\,d}}{(f_2 f_4 + f_1 f_3\,e^{2ik_{2z}d})} \tag{C.35}$$

$$B_1(k_\rho) = i\,\frac{k_\rho\,(g_1 g_4 + g_2 g_3\,e^{2ik_{2z}d})}{k_{1z}\,(g_2 g_4 + g_1 g_3\,e^{2ik_{2z}d})} \tag{C.36}$$

$$B_2(k_\rho) = i\,\frac{\varepsilon_1}{\varepsilon_2}\,\frac{2\,\mu_1\,k_\rho\,g_4}{(g_2 g_4 + g_1 g_3\,e^{2ik_{2z}d})} \tag{C.37}$$

$$B_3(k_\rho) = i\,\frac{\varepsilon_1}{\varepsilon_2}\,\frac{2\,\mu_1\,k_\rho\,g_3\,e^{2ik_{2z}d}}{(g_2 g_4 + g_1 g_3\,e^{2ik_{2z}d})} \tag{C.38}$$

$$B_4(k_\rho) = i\,\frac{\varepsilon_1}{\varepsilon_3}\,\frac{4\,\mu_1\mu_2\,k_\rho\,k_{2z}\,e^{i(k_{2z}-k_{3z})\,d}}{(g_2 g_4 + g_1 g_3\,e^{2ik_{2z}d})} \tag{C.39}$$

$$\begin{aligned} C_1(k_\rho) = \; & 2\,k_\rho^2\,\big[\,(f_4 + f_3\,e^{2ik_{2z}d})\,(g_4 + g_3\,e^{2ik_{2z}d})\,(\varepsilon_1\mu_1 - \varepsilon_2\mu_2) \\ & \qquad + 4\,\varepsilon_1\mu_1\,k_{2z}^2\,(\varepsilon_2\mu_2 - \varepsilon_3\mu_3)\,e^{2ik_{2z}d}\,\big]\;/ \\ & \big[\,(g_2 g_4 + g_1 g_3\,e^{2ik_{2z}d})\,(f_2 f_4 + f_1 f_3\,e^{2ik_{2z}d})\,\big] \end{aligned} \tag{C.40}$$

$$\begin{aligned} C_2(k_\rho) = \; & 2\,k_\rho^2\,\frac{\varepsilon_1}{\varepsilon_2}\,\big[\,f_4\,(g_4 + g_3\,e^{2ik_{2z}d})\,(\varepsilon_1\mu_1 - \varepsilon_2\mu_2) \\ & \qquad - 2\,\mu_1\,k_{2z}\,f_1\,(\varepsilon_2\mu_2 - \varepsilon_3\mu_3)\,e^{2ik_{2z}d}\,\big]\;/ \\ & \big[\,(g_2 g_4 + g_1 g_3\,e^{2ik_{2z}d})\,(f_2 f_4 + f_1 f_3\,e^{2ik_{2z}d})\,\big] \end{aligned} \tag{C.41}$$

$$\begin{aligned} C_3(k_\rho) = \; & 2\,k_\rho^2\,\frac{\varepsilon_1}{\varepsilon_2}\,\big[\,f_3\,(g_4 + g_3\,e^{2ik_{2z}d})\,(\varepsilon_1\mu_1 - \varepsilon_2\mu_2)\,e^{2ik_{2z}d} \\ & \qquad + 2\,\mu_1\,k_{2z}\,f_2\,(\varepsilon_2\mu_2 - \varepsilon_3\mu_3)\,e^{2ik_{2z}d}\,\big]\;/ \\ & \big[\,(g_2 g_4 + g_1 g_3\,e^{2ik_{2z}d})\,(f_2 f_4 + f_1 f_3\,e^{2ik_{2z}d})\,\big] \end{aligned} \tag{C.42}$$

$$\begin{aligned} C_4(k_\rho) = \; & 4\,k_\rho^2\,k_{2z}\,\frac{\varepsilon_1}{\varepsilon_3}\,e^{i(k_{2z}-k_{3z})\,d}\,\big[\,\varepsilon_3\,(g_4 + g_3\,e^{2ik_{2z}d})\,(\varepsilon_1\mu_1 - \varepsilon_2\mu_2) \\ & \qquad + \mu_1\,(f_2 - f_1\,e^{2ik_{2z}d})\,(\varepsilon_2\mu_2 - \varepsilon_3\mu_3)\,\big]\;/ \\ & \big[\,(g_2 g_4 + g_1 g_3\,e^{2ik_{2z}d})\,(f_2 f_4 + f_1 f_3\,e^{2ik_{2z}d})\,\big] \end{aligned} \tag{C.43}$$

In order to stay on the proper Riemann sheet, all square roots

$$k_{jz} = \sqrt{k_j^2 - k_\rho^2} \qquad j \in \{1, 2, 3\} \tag{C.44}$$

have to be chosen such that $\mathrm{Im}\{k_{jz}\} > 0$.

The integrals have to be evaluated numerically. The integration routine has to account for both oscillatory behavior and singularities. It is recommended that the integration range is split into sub-intervals and that the integration path is extended into the complex k_ρ-plane. For some applications it is advantageous to express the Bessel functions J_n in terms of Hankel functions since they converge rapidly for arguments with an imaginary part. An integration routine that proved very reliable is the so-called Gauss-Kronrod routine.