1 Introduction

In near-field scanning optical microscopy (NSOM), a subwavelength sized probe of visible light is used to map the optical properties of a sample with a resolution limited primarily by the tip size and tip to sample separation[1]. NSOM design specifications require that the tip sample distance control should be achieved with sub-nanometer precision. Several distance control mechanisms have been applied to NSOM, including electron tunneling[2,3], capacitance[4], photon tunneling[5], and atomic force[1,6]. The last mechanism detects the atomic force between the tip and sample when they are in very close proximity. Quartz tuning forks have been successfully used as force sensors for tip-sample regulation in NSOM[7,8]. Their very high mechanical quality factor $Q(10^3 - 10^5)$ provides a built-in high gain and makes them very sensitive to sub-pN forces when used at or near their resonance frequency ($10^4$ Hz to $0.5 \times 10^6$ Hz). Compared to optical forces measurement schemes [1,6], their advantage is that the measurement of the oscillation amplitude uses the piezoelectric effect native to quartz crystal, yielding an electric signal proportional to the applied forces and make them small, robust and simple to operate.

This review is organized as follows. In section 2 a model for the quartz tuning fork will be given. In section 3, issues concerning optimal piezoelectric current detection will be addressed. The calibration of the measured piezoelectric signal with the actual amplitude of the fork oscillation at the tip level will also be shown in detail. In section 4 the thermodynamic limit to the signal detection will be discussed as resulting from both the Brownian noise of the tuning fork and the Johnson noise associated with the signal measurement. In section 5, the behavior of the tuning fork in a conventional proportional-integral (PI) closed loop will be presented. It’ll be shown, in particular, that the presently common notion according to which high quality factor Q tuning forks are intrinsically slow for scanning probe microscopy application is wrong. On the contrary, it will be demonstrated that using an optimized PI control loop scanning speed can be increased up to two orders of magnitude at the cost of an expected increase in noise level. The major conclusion is that one should always seek maximum quality factor $Q$ [8].

2 The effective mass harmonic oscillator model

There are two ways to drive a tuning fork. In this section, we will use the one-dimensional effective mass harmonic oscillator model to describe the dynamics of a tuning fork driven mechanically by dither. In general, the resonant dynamics of a tuning fork can be modeled by a harmonic oscillator no matter which driving method is used.

For small amplitudes $u_L(t)$, each arm of the tuning fork can be thought of as a spring with constant $k_{stat}$. Consequently, $u_L(t)$ is the solution of an effective harmonic oscillator equation of motion driven at frequency $\omega$. The equation of motion for such an oscillator is then simply:

$$m_0 \frac{\partial^2 u_L}{\partial t^2} + F_D + k_{stat} u_L = F \exp(i\omega t),$$

(2-1)

where $m_0$ is an effective mass to be determined and $F$ is the mechanical driving force imposed by the dither. $F_D$ is a lossy term proportional to the velocity of the tuning fork. It represents a drag force opposing the movement of the fork arms. We will assume that this drag force is the sum of the forces resulting from the tip sample shear force interaction and from an internal drag force corresponding to the viscous losses occurring in the tuning fork arm. $F_D$ can be characterized by a phenomenological parameter $\gamma$ as:

$$F_D = m_0 \gamma \frac{\partial u_L}{\partial t}.$$  (2-2)

The effective mass $m_0$ is chosen in a way that the resonant frequency $f_0$ of this harmonic oscillator is compatible with the spring constant $k_{stat}$. In the limit of small drag force which corresponds the realistic condition $\gamma \ll \omega_0$, this condition is fulfilled by writing:

$$2\pi f_0 = \omega_0 = \sqrt{\frac{k_{stat}}{m_0}}.$$  (2-3)

We can solve the equation of motion for the tuning fork and we get:

$$u_L(t) = \exp(i\omega t) \frac{F}{m_0}(\omega_0^2 - \omega^2 + i\gamma\omega)^{-1},$$

$$F_D(t) = im_0 \gamma \omega u_L(t).$$  (2-4)
Both $u_L(t)$ and $F_D(t)$ show a resonance peaked at the frequency $f_0$. The amplitude of $u_L(t)$ is given by:

$$\frac{u_L}{u_{L0}} = \gamma \omega_0 \sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2 \omega^2}, \quad (2-5)$$

where $u_{L0} = F/\omega_0 - (ik_{\text{stat}} \gamma)$ is the tip oscillation amplitude at resonance ($\omega = \omega_0$). The above equation shows that the oscillation amplitude of the tip is a Lorenzian function of frequency as shown in Fig. 3_1. And the phase of $u_L(t)$ is given by:

$$\theta = \arctan \left( \frac{\gamma \omega}{\omega_0^2 - \omega^2} \right). \quad (2-6)$$

At this point, we define the quality factor ($Q$ factor) as

$$Q_0 = f_0 / \Delta f, \quad (2-7)$$

where $\Delta f$ is the full width at half maximum (FWHM) of the Lorenzian shaped resonance of the tip oscillation amplitude. On resonance the tip amplitude and the drag force experienced by the tuning fork is given by

$$u_{L0} = \frac{\sqrt{3}Q_0 F}{ik_{\text{stat}}}, \quad (2-8)$$

$$F_{D0} = i \frac{k_{\text{stat}}}{\sqrt{3}Q_0 u_{L0}}$$

So we see that for small losses, the maximum vibration amplitude $u_{L0}$ is proportional to $Q_0$. The local potential on the tuning fork due to the piezoelectric effect is proportional to $u_L(t)$. Therefore, a tuning fork can be used as a sensitive drag force detector.

The optical fiber with its tapered tip glued along the side of one of the arms of a quartz crystal tuning fork. The tip at the end of the optical fiber protrudes out of the arm on which it is attached. The tip is hard enough that it doesn’t vibrate by itself, instead the tip oscillates with the tuning fork. The fiber tip acts as a shear force pick-up. The tuning fork is used as a force sensor. The tuning fork is vibrated in such a way that the tip oscillates parallel to the sample surface. Both arms of the fork are piezo-electrically coupled through the metallic contact pads. The geometry of the contacts and the coupling between the two arms insures that only one resonant vibration mode of the fork is excited. ON resonance, the bending amplitude of the prongs is maximum. Tuning fork is a sensitive sensor for distance control. Whenever the distance between the sample and the tip is changed, the tuning fork’s resonance frequency shift is instantaneous; while the amplitude of the tuning fork oscillation shift is lowest due to relaxation time delay but the sensitivity is the best. And phase shift is in between the above two methods. After all, we can use the voltage output from the tuning fork with phase locked loop circuit as feedback to keep the distance between the sample surface and the tuning fork as a constant.

If we drive the tuning fork with a resonant voltage, then the induced current is:

$$I = j2\pi f_0 \left( c_i u_L + c_p U_{\text{excitation}} \right), \quad (2-9)$$

where $f_0$ is the resonance frequency; $c_i$ is a constant which is related to properties and geometrical shape of the tuning fork; $c_p$ is capacitance of the tuning fork. The first term of the above equation contains the shear force and its amplitude is large only near resonance; the second term is responsible for the capacitance background signal. We can’t reduce the drive voltage $U_{\text{excitation}}$ to reduce the background signal because the tuning fork motion amplitude $u_L$ is proportional to $U_{\text{excitation}}$. However, we can reduce the coupling capacitance $c_p$.

### 3 Signal detection and its calibration

The key to implementing tuning forks for force detection is to accurately measure the fundamental resonance of the tuning fork as a function of applied force. This can be done either by shaking the fork at its mechanical resonance with a dither and monitoring the induced voltage, or by directly driving the tuning fork with a resonant voltage and measuring the induced current. Both schemes have advantages and disadvantages. We here discuss how to implement the later, since it provides for a system that is mechanically simpler at the expense of only minimal electronics.

When the tuning fork is driven with an external oscillating voltage $U_{\text{exc}}$, the signal to be monitored is the corresponding ac current flowing through the tuning fork. At frequencies away from the resonant condition, the system response is nothing more than a capacitor $C_p$ (typically a few pF for commercial 33 kHz quartz forks). The resulting current is $I_{\text{capa}} = i2\pi fC_p U_{\text{exc}}$ which increases linearly with the frequency. At resonance, the current peaks to a value $I_{\text{max}}$, which depends on the drag force experienced by the fork. This useful signal is interfering with the capacitive stray background signal $I_{\text{capa}}$. The total current flowing through the tuning fork can be written as

$$I = i2\pi f_0 \left( 3d_{11}E \frac{T W}{L} x_L + c_p U_{\text{exc}} \right), \quad (2-10)$$

where $d_{11}$ is the longitudinal piezoelectric modulus, $E$ is the elasticity modulus (i.e. Young’s modulus) of the fork material, $T$ and $W$ are the width and height of the cross section of one tuning fork arm, respectively, and $L$ is the length of one arm of the tuning fork. The first term in the bracket is due to the shear force, its amplitude is large only around resonant conditions. The second term is the capacitive ground signal. Since $x_L$ is
proportional to $U_{\text{exc}}$, one cannot reduce the drive voltage $U_{\text{exc}}$ in order to reduce the background signal.

To eliminate the effect of the package capacitance, a bridge circuit was used (Fig 3_1a). The transformer yields two wave forms phase shifted from each other by 180°. By appropriately adjusting the variable capacitor, the current through the package capacitance is negated by the current through the variable capacitor. A standard operational amplifier was used for current-to-voltage conversion. The current to voltage gain of the circuit has been calibrated for dc to 100 kHz and was found to be described as

$$Z_{\text{gain}} = R_g / \sqrt{1 + (2\pi f R_g C_g)^2} \text{ where } f \text{ is the frequency,}$$

$$R_g = 9.51 \text{M}\Omega, \text{ and } C_g = 0.260 \text{pF is the stray capacitance in parallel with } R_g.$$ 

Having calibrated the measurement system, the impedance of the tuning fork can be accurately measured. The ratio of the output to input for a bare fork is shown as the closed circles in Fig 3_2b. The response fits well to the Lorentzian line shape

$$Af_0 f / Q / \sqrt{(f_0^2 - f^2)^2 + (f_0 f / Q)^2} \quad (3-1)$$

with $\Lambda=17.14$, $f_0=32733.3$kHz, and $Q=8552$, shown as the solid line. This result demonstrates that effect of the package capacitance has been eliminated and that the harmonic oscillator is an excellent model for the response of the tuning fork. Values for the RLC circuit mode can be determined by comparing the above-mentioned line shape to the formula for the gain of the amplifier with the RLC resonator as the input impedance, yielding $R = Z_{\text{gain}} / A = 0.494 \text{M}\Omega$.

$L = RQ / 2\pi f_0 = 20.5$kHz, and $C = 1 / (4\pi^2 f_0^2 L) = 1.14$F. $R$ is large enough that it’s attributed completely to mechanical dissipation associated with motion of the quartz.

Another important calibration parameter is the amplitude of oscillation of the tuning fork as a function of the output voltage, which was denoted by the parameter $\alpha$. This has been determined by interferometrically measuring the physical amplitude of oscillation of one arm of the tuning fork while simultaneously measuring the output voltage of the system. This interferometric technique is described in detail in Ref 10. To interpret this measurement, it is necessary to define the relationship between the measured output voltage and the oscillation amplitude of one arm of the tuning fork. The output voltage is sensitive only to the antisymmetric mode of the tuning fork, $V_{out} = c(x_1 - x_2)$, where $c$ is a constant and $x_1$ and $x_2$ are the amplitude of motion of the two arms of the fork. When driving the fork with an external voltage, only the antisymmetric mode is excited, yielding $x_1 = -x_2$. Thus, we define $V_{out} = 2cx_1 = x_1 / \alpha$. The calibration yields $\alpha = 59.6 \pm 0.1$ pm/mV. Viewing the tuning fork as a current source, it is convenient to write the above-mentioned equation in terms of the current to voltage converting resistor, $\alpha = \beta / Z_{\text{gain}}$, yielding $\beta = 50.505$ mA. Because the charge separation in the tuning fork is amplitude dependent, calibration in terms of current yields an accurate measure of the amplitude of the tuning fork oscillation.

4 Thermodynamic limits to force detection

The fundamental measurement limit of the ability to measure the resonance is dictated by the intrinsic system noise. These limits can be determined experimentally by measuring the output noise with the input grounded. The resulting output is shown in Fig 4_1 as the filled circles. A noise analysis of the circuit shows that the two primary noise sources are Johnson noise of the feedback resistor, $\sqrt{4k_BT R} V / \sqrt{Hz}$, and Johnson noise associated with mechanical dissipation in the fork, as manifested by the $R$ in the series RLC equivalent circuit, $\sqrt{4k_BT R (Z_{\text{gain}} / R)}$

$$\times((ff_0 / Q) / \sqrt{(f_0^2 - f^2)^2 + (f_0 f / Q)^2} V / \sqrt{Hz},$$

where $k_B$ is the Boltzmann constant and $T$ is the temperature. All other parameters have already been determined experimentally. These two noise terms add in quadrature. It is shown as a solid line in Fig 4_1.
Fig 4.1 The noise spectrum of the measurement circuit in Fig 3.1a [from Ref 8].

The power spectrum of the noise associated with the tuning fork can be integrated so as to get the root mean square (rms) voltage noise,

\[ V_{\text{rms}}^2 = 4k_B T R (Z_{\text{gain}} / R)^2 (\pi f_0 / 2Q) \]

which evaluates to \( V_{\text{rms}} = 3.81 \mu \text{V} \) at room temperature. This can be related to the thermal motion of the arms of the tuning fork by taking the time average of the previous definition relating the output voltage to the motion of the arm of the fork

\[ V_{\text{rms}}^2 = \left\{ c(x_1 - x_2)^2 \right\} = c^2 \left\{ \langle x_1^2 \rangle + \langle x_2^2 \rangle - 2\langle x_1 x_2 \rangle \right\} = 2c^2 \langle x_1^2 \rangle \]

where it is assumed that the arms are only very weakly coupled and that their motion is uncorrelated. The interferometric measurement allows one to convert this to an rms displacement of the arm, \( x_{\text{rms}} = \sqrt{2} a V_{\text{rms}} = 0.32 \text{pm} \). This is the random motion of one arm of the fork due to thermal fluctuations. With this value for \( x_{\text{rms}} \), an effective spring constant can be calculated via the equipartition theorem, \( K = k_B T / x_{\text{rms}}^2 \approx 40.2 \text{kN/m} \). This value agrees well with the theoretically determined spring constant from the formula \( K = EWT^3 / (4L^3) \), where \( E \) is Young’s modulus of the fork material, \( W \) is the width of one arm of the fork, \( T \) is the length of one arm of the fork, and \( L \) is the height of the cross section of one arm of the fork, respectively.

The thermal energy can be thought of in terms of an effective force acting on the tuning fork. This force has a flat power spectrum, \( S_f \), in units of \( N^2 / Hz \). One can calculate the power spectrum from the equation:

\[ x_{\text{rms}}^2 = \int_0^\infty S_f \left| \frac{f_0^2 / K}{f_0^2 - f^2 - i(f_0/Q)} \right|^2 df \]

Evaluating the integral and again using the equipartition theorem, one can obtain \( S_f^{1/2} = \sqrt{2/\pi} f_0 Q (k_B T / 2x_{\text{rms}}) = 0.44 \text{pN/Hz} \).

The ratio of signal voltage to noise voltage, \( S/N \), as a function of bandwidth, \( \Delta f \), is given by

\[
\frac{S}{N} = \frac{QF / (\beta K)}{\sqrt{4k_B T (\Delta f / R_g + Q / Z_r) \int df \frac{(f_0/Q)^2}{(f_0^2 - f^2)^2 + (f_0/Q)^2}}}
\]

where all terms are written as currents, \( R \) has been written as \( R_g / \sqrt{R_g} \). The noise is dominated by the resonance impedance of the tuning fork as long as the resonance impedance is significantly smaller than \( R_g \). This is true for

\[
\Delta f \leq \frac{f_0}{Q} \sqrt{QR_g / Z_r - 1}.
\]

At larger bandwidths, the noise associated with \( R_g \) becomes dominant. This can be remedied by using larger \( R_g \); however, it degrades the time response of the amplifier due to the stray capacitance. So, the choice of \( R_g \) is a trade-off between fast response and better S/N ratio.

5 Measurement speed

The final issue is the speed with which the measurement can follow changes in force. Scanning images of surfaces are made by fixing the height of the probe above the surface using a closed loop control system. Some combination of the amplitude, and/or phase of the tuning fork signal is used as the set point in the control loop. When using conventional Si cantilevers it is well known that one must resort to phase sensitive techniques of system control in order to make the system respond faster than of the open loop response time of the resonator, \( 1 / \tau = \pi f_0 / Q \). These techniques have also been successfully implemented with tuning fork systems.[11] However, in the following it’s demonstrated that conventional proportional and integral (PI) feedback control is sufficient for most uses of tuning forks.

The experiment is done at room temperature, pressure, and atmosphere. The setup is shown schematically in Fig 5.1. A tapered fiber probe is mounted on the fork consistent with operation in a near-field scanning optical microscope. The open loop response of the loaded tuning fork corresponds to the line shape in Eq. (3-1) with \( f_0 / 2Q = 8 \text{Hz} \) and correspondingly \( \tau \approx 20 \text{ms} \). The tuning fork is driven at resonance by a voltage source of fixed amplitude and frequency and the resulting current is measured with the circuit of Fig 3.1a. This output is detected with a lock-in amplifier whose phase is referenced to the phase of the resonantly driven tuning fork. The “X” output of the lock-in amplifier is fed to conventional PI feedback electronics which then adjusts the position of the probe above the surface.
The set point of the feedback is adjusted so that the probe is fixed approximately 10 nm above the surface. To test the response of the tracking system, the vertical position of the sample is intentionally dithered at 100 Hz using a 1 nm square wave. The output of the control electronics, shown in Fig 5_2, clearly follows the 100 Hz signal. The rise time of the response is of order 0.5 ms, 40 times faster than the open loop response. It’s clear that simple PI control electronics yield a stable tracking system that can operate at speeds much faster than the response of the tuning fork resonator.

Some comments: As we can see from Fig 5_2, the signal is very noisy. In fact, power spectrum of this signal shows a large noise at frequency $1/\tau$ (i.e., the open loop response frequency). This reason has prevented one from realizing fast scanning speed without reducing the Q factor of the tuning fork.

References