VON FREQUENZ- ZU ZEITBEREICH

$$\mathbf{A}(\mathbf{r}) \;=\; \mu_0 \int_V G_0(\mathbf{r},\mathbf{r}')\; \mathbf{j}_0(\mathbf{r}')\; dV'$$

$$\mathbf{j}_0(\mathbf{r},t) \;=\; \int_{-\infty}^{\infty} \hat{\mathbf{j}}_0(\mathbf{r},\omega)\, \mathrm{e}^{-\imath\omega t}\, \mathrm{d}\omega \qquad \mathbf{A}(\mathbf{r},t) \;=\; \int_{-\infty}^{\infty} \hat{\mathbf{A}}(\mathbf{r},\omega)\, \mathrm{e}^{-\imath\omega t}\, \mathrm{d}\omega$$

$$\hat{\mathbf{A}}(\mathbf{r},\omega) \;=\; \mu_0 \int_V \hat{G}_0(\mathbf{r},\mathbf{r}',\omega)\; \hat{\mathbf{j}}_0(\mathbf{r}',\omega)\; dV'$$

$$\mathbf{A}(\mathbf{r},t) \;=\; \frac{\mu_0}{2\pi} \int_V G_0(\mathbf{r},\mathbf{r}',t) \;*\; \mathbf{j}_0(\mathbf{r}',t)\; dV'$$

BELIEBIGE ZEITABHÄNGIGKEIT

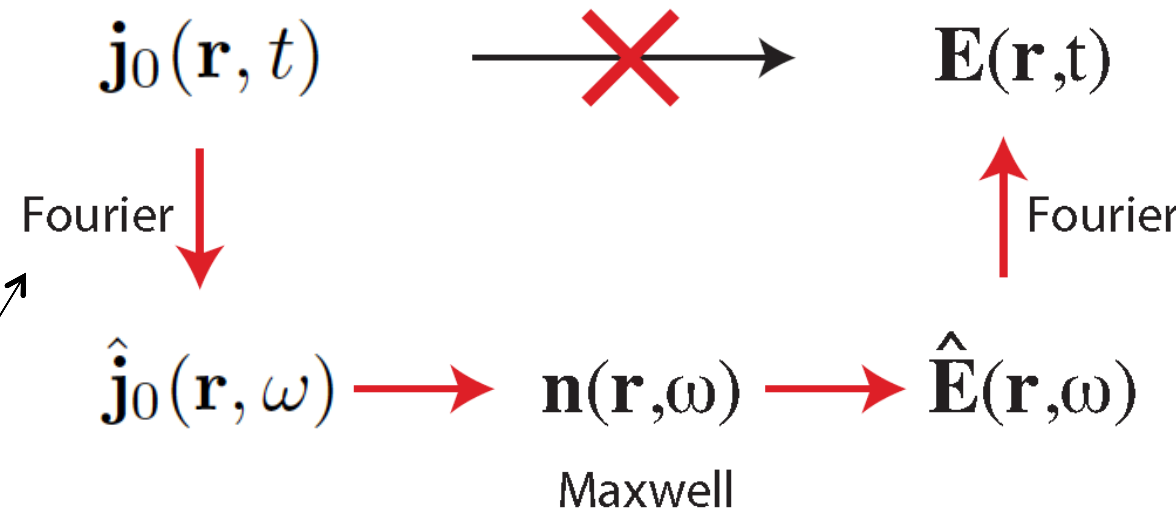
$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int_V \frac{\mathbf{j}_0(\mathbf{r}', t - |\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}\, dV'$$

$$\phi(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho_0(\mathbf{r}', t - |\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}\, dV'$$

$$\mathbf{E}(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t) - \nabla\phi(\mathbf{r},t)$$

$$\mathbf{B}(\mathbf{r},t) = \nabla \times \mathbf{A}(\mathbf{r},t)$$

DISPERSIVE MEDIEN



$$\hat{\mathrm{j}}_0(\mathbf{r},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{j}_0(\mathbf{r},t)\,\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}t$$

$$\mathbf{E}(\mathbf{r},t) = \int_{-\infty}^{\infty} \hat{\mathrm{E}}(\mathbf{r},\omega)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}\omega$$

$$\hat{\mathrm{E}}(\mathbf{r},\omega) = i\omega\mu_0\mu(\omega)\int_V \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}',\omega)\,\hat{\mathrm{j}}_0(\mathbf{r}',\omega)\,dV'$$

DIPOLFELDER

$$\hat{E}_\vartheta(\mathbf{r},\omega) \;=\; \hat{p}(\omega)\,\frac{\sin\vartheta}{4\pi\varepsilon_0\varepsilon(\omega)}\,\frac{\exp[\mathrm{i}k(\omega)\,r]}{r}\,k^2(\omega)\left[\frac{1}{k^2(\omega)\,r^2}-\frac{\mathrm{i}}{k(\omega)\,r}-1\right]$$

$$k=\omega/c \quad \varepsilon=1 \qquad \longrightarrow \qquad E^f_\vartheta(\mathbf{r},t) \;=\; \int_\infty^\infty \hat{E}^f_\vartheta(\mathbf{r},\omega)\,\mathrm{e}^{-i\omega t}\,d\omega$$

$$= -\frac{\sin\vartheta}{4\pi\varepsilon_0}\frac{1}{c^2\,r}\int_\infty^\infty \omega^2\,\hat{p}(\omega)\,\mathrm{e}^{-i\omega(t-r/c)}\,d\omega$$

$$= \frac{\sin\vartheta}{4\pi\varepsilon_0}\frac{1}{c^2\,r}\;\frac{d^2\,p(\tau)}{d\tau^2}\bigg|_{\tau=t-r/c}$$

$$E_r(t) \;=\; \frac{\cos\vartheta}{4\pi\varepsilon_0}\left[\frac{2}{r^3}+\frac{2}{c\,r^2}\frac{\mathrm{d}}{\mathrm{d}\tau}\right]p(\tau)\,\bigg|_{\tau=t-r/c}$$

$$E_\vartheta(t) \;=\; -\frac{\sin\vartheta}{4\pi\varepsilon_0}\left[\frac{1}{r^3}+\frac{1}{c\,r^2}\frac{\mathrm{d}}{\mathrm{d}\tau}+\frac{1}{c^2 r}\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\right]p(\tau)\,\bigg|_{\tau=t-r/c}$$

$$H_\varphi(t) \;=\; -\frac{\sin\vartheta}{4\pi\varepsilon_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[\frac{1}{c\,r^2}\frac{\mathrm{d}}{\mathrm{d}\tau}+\frac{1}{c^2 r}\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\right]p(\tau)\,\bigg|_{\tau=t-r/c}$$

LORENTZ'SCHES ENERGIESPEKTRUM

Gedämpfter harmonischer Oszillator: $\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{p}(t) + \gamma_0 \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{p}(t) + \omega_0^2 \mathbf{p}(t) = 0$

Ansatz: $\mathbf{p}(t) = \mathrm{Re}\left\{\mathbf{p}_0 \, \mathrm{e}^{-\mathrm{i}\omega_0 \overbrace{\sqrt{1-(\gamma_0^2/4\omega_0^2)}}^{\approx 1}\, t} \, \mathrm{e}^{\gamma_0 t/2}\right\}$

Elektrisches Fernfeld: $\hat{E}_\vartheta(\omega) = \frac{1}{2\pi}\int_{r/c}^{\infty} E_\vartheta(t)\,\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}t$

↑ Dipol fängt t=0 an zu oszillieren

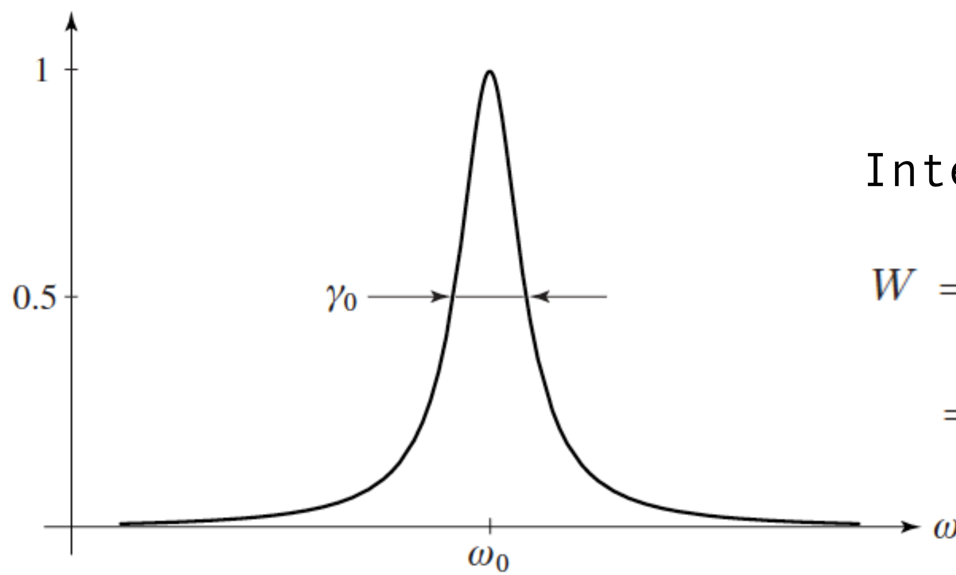
$$\rightarrow \quad \hat{E}_\vartheta(\omega) = \frac{1}{2\pi}\frac{|\mathbf{p}|\,\sin\vartheta\,\omega_0^2}{8\pi\varepsilon_0 c^2 r}\left[\frac{\exp(\mathrm{i}\omega r/c)}{\mathrm{i}(\omega+\omega_0)-\gamma_0/2} + \frac{\exp(\mathrm{i}\omega r/c)}{\mathrm{i}(\omega-\omega_0)-\gamma_0/2}\right]$$

LORENTZ'SCHES ENERGIESPEKTRUM

Energie pro Einheitswinkel $\mathrm{d}\Omega = \sin\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\varphi$:

$$\frac{\mathrm{d}W}{\mathrm{d}\Omega} = \int_{-\infty}^{\infty} I(\mathbf{r},t)\, r^2 \, \mathrm{d}t \;=\; r^2 \sqrt{\frac{\varepsilon_0}{\mu_0}} \int_{-\infty}^{\infty} |E_\vartheta(t)|^2 \, \mathrm{d}t \;=\; 2\pi r^2 \sqrt{\frac{\varepsilon_0}{\mu_0}} \int_{-\infty}^{\infty} |\hat{E}_\vartheta(\omega)|^2 \, \mathrm{d}\omega$$

$$\rightarrow \quad \boxed{\frac{\mathrm{d}W}{\mathrm{d}\Omega\,\mathrm{d}\omega} = \frac{1}{4\pi\varepsilon_0} \frac{|\mathbf{p}|^2 \sin^2\vartheta \, \omega_0^4}{4\pi^2 c^3 \gamma_0^2} \left[\frac{\gamma_0^2/4}{(\omega-\omega_0)^2 + \gamma_0^2/4} \right]}$$



Integriert :

$$W = \frac{|\mathbf{p}|^2}{4\pi\varepsilon_0} \frac{\omega_0^4}{3\,c^3\,\gamma_0}$$

$$= \bar{P} / \gamma_0$$