# ENERGIE- UND IMPULSERHALTUNG

$$\underbrace{\mathbf{H}\cdot(\nabla\times\mathbf{E})-\mathbf{E}\cdot(\nabla\times\mathbf{H})}_{\nabla\cdot(\mathbf{E}\times\mathbf{H})} = -\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}-\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}-\mathbf{j}_0\cdot\mathbf{E}$$

$$\boxed{\int_{\partial V}(\mathbf{E}\times\mathbf{H})\cdot\mathbf{n}\,da = -\int_V\left[\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}+\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}+\mathbf{j}_0\cdot\mathbf{E}\right]dV}$$

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John Poynting / Oliver Heaviside (1883) :

$$\int_{\partial V}\overbrace{(\mathbf{E}\times\mathbf{H})}^{\mathbf{S}}\cdot\mathbf{n}\,da + \frac{\partial}{\partial t}\int_V\overbrace{\frac{1}{2}\left[\mathbf{D}\cdot\mathbf{E}+\mathbf{B}\cdot\mathbf{H}\right]}^{W}dV =$$

$$\underbrace{-\int_V\mathbf{j}_0\cdot\mathbf{E}\,dV-\frac{1}{2}\int_V\left[\mathbf{E}\cdot\frac{\partial\mathbf{P}}{\partial t}-\mathbf{P}\cdot\frac{\partial\mathbf{E}}{\partial t}\right]dV-\frac{\mu_0}{2}\int_V\left[\mathbf{H}\cdot\frac{\partial\mathbf{M}}{\partial t}-\mathbf{M}\cdot\frac{\partial\mathbf{H}}{\partial t}\right]dV}_{\text{Verluste / Quellen}}$$

# ERHALTUNGSSÄTZE

Ladung :

$$\int_{\partial V} \mathbf{j} \cdot \mathbf{n} \, da + \frac{\partial}{\partial t} \int_V \rho \, dV = 0$$

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Energie :

$$\int_{\partial V} \mathbf{S} \cdot \mathbf{n} \, da + \frac{\partial}{\partial t} \int_V w \, dV = -\int_V \mathbf{j} \cdot \mathbf{E} \, dV \quad + \text{NL}$$

$\mathbf{S} = [\mathbf{E} \times \mathbf{H}]$

$w = \frac{1}{2}\left[\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}\right]$

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# MONOCHROMATISCHE FELDER

Zeitliches Mittel:

$$\int_{\partial V} \langle \mathbf{S}(\mathbf{r}) \rangle \cdot \mathbf{n} \, da \;=\; -\frac{1}{2} \int_V \mathrm{Re} \{ \mathbf{j}^*(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r}) \} \; dV$$

Zeitgemittelter Poynting Vektor :

$$\langle \mathbf{S}(\mathbf{r}) \rangle \;=\; \frac{1}{2} \mathrm{Re} \{ \mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r}) \}$$

Annahme :

Polarisierungs- und Magnetisierungsströme sind verlustfrei

# INTENSITÄT

$$I(\mathbf{r}) = |\langle \mathbf{S}(\mathbf{r}) \rangle|$$

Leistung pro Fläche

Fernfeld (ebene Wellen) :

$$\langle \mathbf{S}(\mathbf{r}) \rangle \;=\; \frac{1}{2}\frac{1}{Z_i}\,|\mathbf{E}(\mathbf{r})|^2\,\mathbf{n}_r$$

Gesamtleistung :

$$\bar{P} \;=\; \int_{\partial V} \langle \mathbf{S}(\mathbf{r}) \rangle \cdot \mathbf{n}\; da \;=\; \int_{\partial V} I(\mathbf{r})\; da$$

# ENERGIEDICHTE

$$\int_{\partial V} \mathbf{S} \cdot \mathbf{n}\, da + \frac{\partial}{\partial t} \int_V w\, dV = -\int_V \mathbf{j} \cdot \mathbf{E}\, dV \quad + \text{NL}$$

$\mathbf{S} \rightarrow [\mathbf{E} \times \mathbf{H}]$

$w \rightarrow \frac{1}{2}\left[\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}\right]$

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Mittlere Energiedichte: $\bar{w} = \frac{1}{2}\langle \mathbf{D} \cdot \mathbf{E} \rangle + \frac{1}{2}\langle \mathbf{B} \cdot \mathbf{H} \rangle$

$\mathbf{D} \rightarrow \mathrm{Re}\{\, \mathbf{D}(\mathbf{r}) \exp[-i\omega t]\,\}$

$$\bar{w} = \frac{1}{4}\mathrm{Re}\{\mathbf{D}(\mathbf{r}) \cdot \mathbf{E}^*(\mathbf{r})\} + \frac{1}{4}\mathrm{Re}\{\mathbf{B}(\mathbf{r}) \cdot \mathbf{H}^*(\mathbf{r})\}$$

$\mathbf{D}(\mathbf{r}) \rightarrow \varepsilon_0 \varepsilon(\omega) \mathbf{E}(\mathbf{r})$

$$\bar{w} = \frac{\varepsilon_0 \varepsilon(\omega)}{4} |\mathbf{E}(\mathbf{r})|^2 + \frac{\mu_0 \mu(\omega)}{4} |\mathbf{H}(\mathbf{r})|^2$$

$$= \frac{\varepsilon_0 \varepsilon(\omega)}{2} \langle \mathbf{E} \cdot \mathbf{E} \rangle + \frac{\mu_0 \mu(\omega)}{2} \langle \mathbf{H} \cdot \mathbf{H} \rangle$$

# ENERGIEDICHTE IN DISPERSIVEN MEDIEN

$$\int_{\partial V} (\mathbf{E} \times \mathbf{H}) \cdot \mathbf{n} \, da = - \int_V \left[ \mathbf{H} \cdot \frac{\partial \mathrm{B}}{\partial t} + \mathbf{E} \cdot \frac{\partial \mathrm{D}}{\partial t} + \mathbf{j} \cdot \mathbf{E} \right] dV$$

$$\mathbf{E}(t') = \int \hat{\mathbf{E}}(\omega) \exp[-\mathrm{i}\omega t'] \, \mathrm{d}\omega \qquad \mathbf{D}(t') = \int \hat{\mathbf{D}}(\omega) \exp[-\mathrm{i}\omega t'] \, \mathrm{d}\omega$$

$$\longrightarrow \quad w_E(\mathbf{r}, t) = \varepsilon_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\omega' \, \varepsilon^*(\omega')}{\omega' - \omega} \, \hat{\mathbf{E}}(\omega) \cdot \hat{\mathbf{E}}^*(\omega') \, \mathrm{e}^{i(\omega' - \omega)t} \, d\omega' \, d\omega$$

Slowly varying envelope approximation:

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}\{\tilde{\mathbf{E}}(\mathbf{r}, t)\} = \mathrm{Re}\{\mathbf{E}_0(\mathbf{r}, t) \, \mathrm{e}^{-i\omega_0 t}\} \qquad \longrightarrow \quad \bar{w_E}(\mathbf{r}) = \frac{\varepsilon_0}{4} \left. \frac{d\,[\omega \, \varepsilon'(\omega)]}{d\omega} \right|_{\omega = \omega_0} |\mathbf{E}_0(\mathbf{r})|^2$$

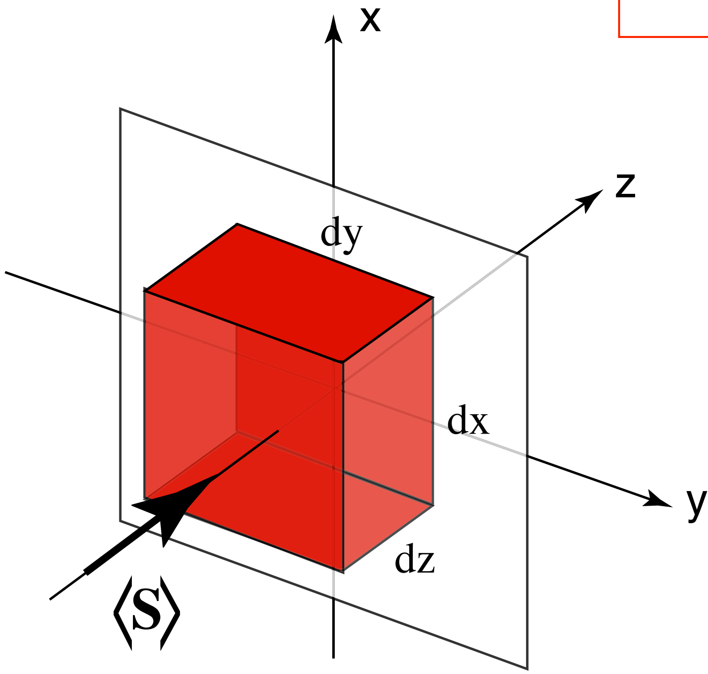
$$\bar{W} = \left[ \varepsilon_0 \frac{d\,[\omega \, \varepsilon'(\omega)]}{d\omega} \left\langle \mathbf{E} \cdot \mathbf{E} \right\rangle + \mu_0 \frac{d\,[\omega \, \mu'(\omega)]}{d\omega} \left\langle \mathbf{H} \cdot \mathbf{H} \right\rangle \right]$$

# STRAHLUNGSDRUCK (III)

$$\langle \mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t) \rangle \;=\; \frac{1}{2}\mathrm{Re}\{\mathbf{E}(\mathbf{r}) \times \mathbf{H}^{*}(\mathbf{r})\} \;=\; \langle \mathbf{S}(\mathbf{r}) \rangle$$

$$\rightarrow \quad \boxed{\mathbf{p}(\mathbf{r}) \;=\; \frac{1}{c^2} \int_V \langle \mathbf{S}(\mathbf{r}) \rangle \, dV}$$

Elektromagnetischer Impuls



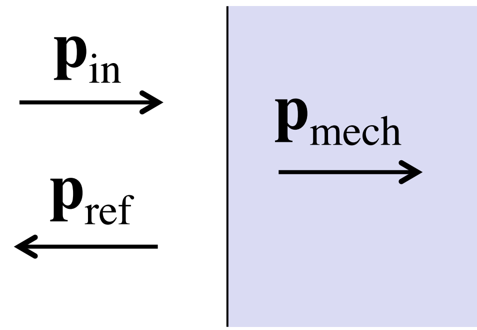

Kraft = Impuls der die Fläche (dx dy) pro Zeiteinheit dt durchdringt

$$\frac{d\mathbf{p}}{dt} \;=\; \frac{1}{c^2} \langle \mathbf{S} \rangle \, dx\, dy \frac{dz}{dt} \longrightarrow c$$

Strahlungsdruck : $\boxed{\mathbf{P} \;=\; \frac{1}{c} \langle \mathbf{S} \rangle}$

# STRAHLUNGSDRUCK (IV)

Impulserhaltung :




Druck auf reflektierende Fläche: $\mathbf{P}_{\mathrm{in}} = \mathbf{P}_{\mathrm{ref}} + \mathbf{P}_{\mathrm{mech}}$

$$\mathbf{P}_{\mathrm{in}} \rightarrow \frac{1}{c}\langle \mathbf{S} \rangle \qquad \mathbf{P}_{\mathrm{ref}} \rightarrow -R\frac{1}{c}\langle \mathbf{S} \rangle$$

$$\rightarrow \boxed{\mathbf{P}_{\mathrm{mech}} = [1+R]\,\frac{1}{c}\langle \mathbf{S} \rangle} \qquad R = |r|^2$$

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Energie :
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Impuls :
$$\int_{\partial V} \overleftrightarrow{\mathbf{T}} \cdot \mathbf{n}\, da - \frac{\partial}{\partial t} \int_V \mathbf{g}_{\text{field}}\, dV = \int_V \frac{\partial}{\partial t} \mathbf{g}_{\text{mech}}\, dV$$

$\overleftrightarrow{\mathbf{T}} \rightarrow \varepsilon_0 \mathbf{E}\mathbf{E} + \mu_0 \mathbf{H}\mathbf{H} - \frac{1}{2}\left(\varepsilon_0 E^2 + \mu_0 H^2\right) \overleftrightarrow{\mathbf{I}}$

$\mathbf{g}_{\text{field}} \rightarrow \frac{1}{c^2}[\mathbf{E} \times \mathbf{H}]$

$\frac{\partial}{\partial t}\mathbf{g}_{\text{mech}} \rightarrow [\rho \mathbf{E} + \mathbf{j} \times \mathbf{B}]$

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Drehmoment :
$$\int_{\partial V} \left[\overleftrightarrow{\mathbf{T}} \times \mathbf{r}\right] \cdot \mathbf{n}\, da + \frac{\partial}{\partial t} \int_V \mathbf{j}_{\text{field}}\, dV = -\int_V \frac{\partial}{\partial t} \mathbf{j}_{\text{mech}}\, dV$$

$\mathbf{j}_{\text{field}} \rightarrow \frac{1}{c^2} \mathbf{r} \times [\mathbf{E} \times \mathbf{H}]$

$\frac{\partial}{\partial t}\mathbf{j}_{\text{mech}} \rightarrow \mathbf{r} \times [\rho \mathbf{E} + \mathbf{j} \times \mathbf{B}]$