

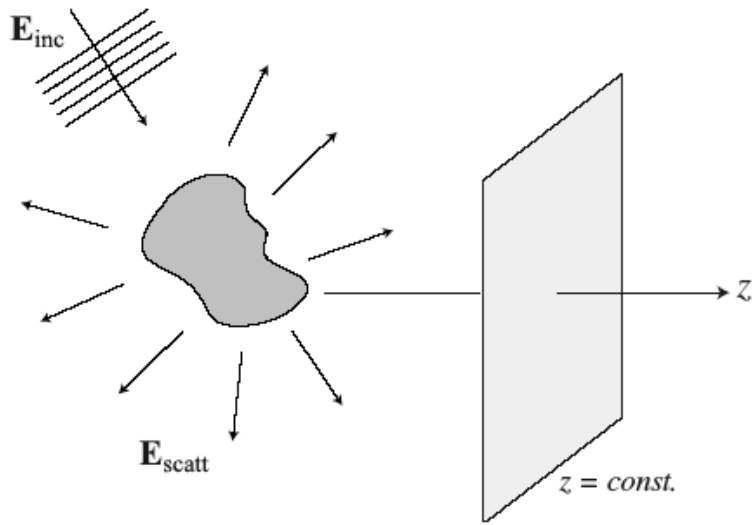
Heute im Programm

- Wiederholung: Feldwinkelspektrum
- Wiederholung: Gauss'scher Strahl
- Wiederholung: Fernfeldnäherung

- Fourieroptik

- Party-Party/Schneemann-Brille

Das Feldwinkelspektrum



Feldwinkelspektrum:

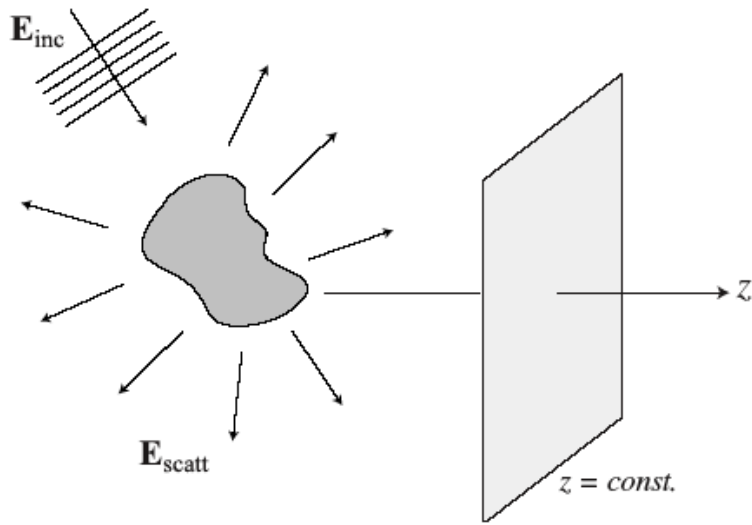
$$\hat{\mathbf{E}}(k_x, k_y; z) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \mathbf{E}(x, y, z) e^{-i[k_x x + k_y y]} dx dy$$

Durch inverse Transformation:

$$\mathbf{E}(x, y, z) = \iint_{-\infty}^{\infty} \hat{\mathbf{E}}(k_x, k_y; z) e^{i[k_x x + k_y y]} dk_x dk_y$$

Enthält das Feldwinkelspektrum evaneszente Komponenten?

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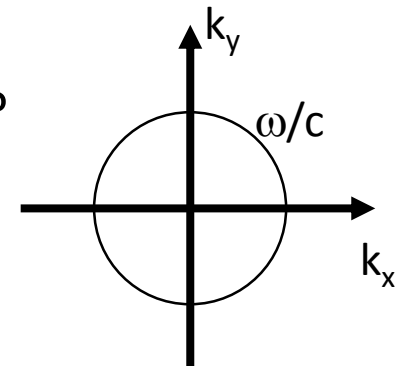
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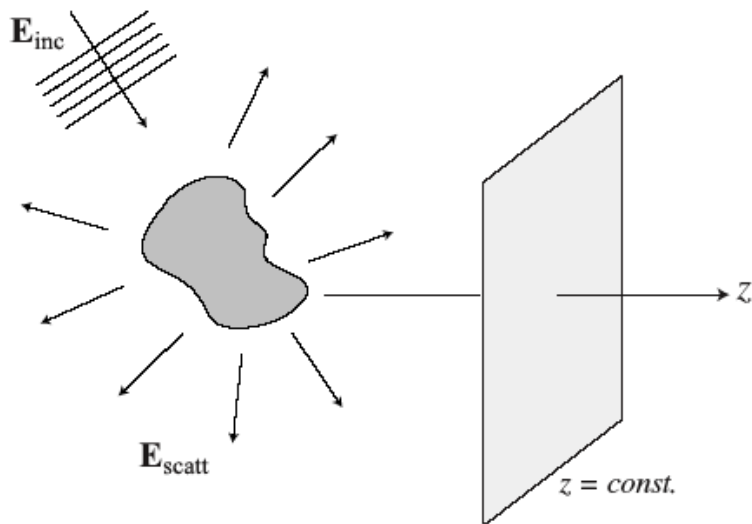
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Wo liegen diese in der k-Ebene?



Das Feldwinkelspektrum



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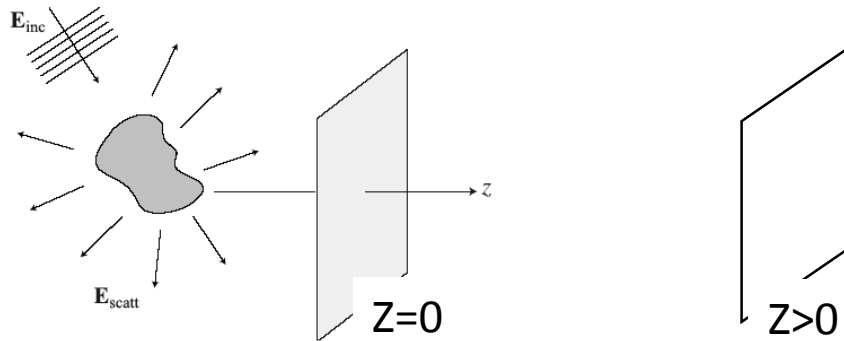
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$$k_z \equiv \sqrt{(k^2 - k_x^2 - k_y^2)} \quad \text{with} \quad \text{Im}\{k_z\} \geq 0$$

Ausbreitung von Feldern im homogenen Medium



$$\longrightarrow \underbrace{\hat{\mathbf{E}}(k_x, k_y; z)}_{\text{Bild-Ebene}} = \underbrace{\hat{H}(k_x, k_y; z)}_{\text{Transfer-Funktion}} \underbrace{\hat{\mathbf{E}}(k_x, k_y; 0)}_{\text{Quellen-Ebene}}$$

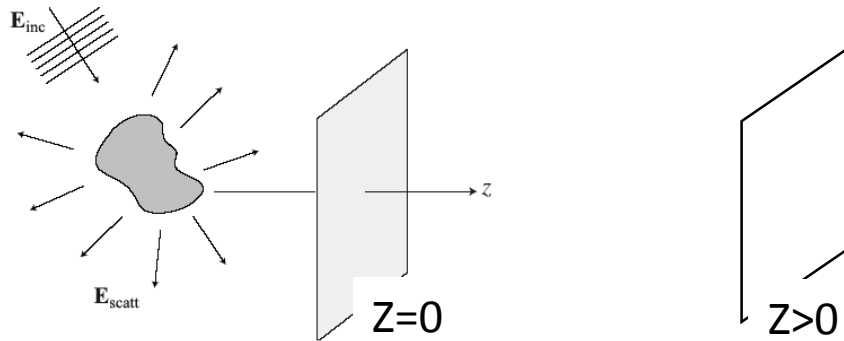
Transfer-Funktion (Filter in k_x, k_y Raum) :

$$\hat{H}(k_x, k_y; z) = \text{[Redacted]}$$

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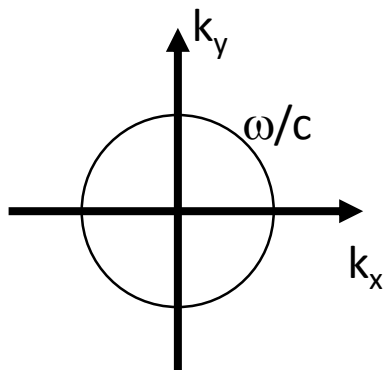
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Transfer-Funktion (Filter in k_x, k_y Raum) :

$$\hat{H}(k_x, k_y; z) = e^{\pm i k_z z}$$



→ pass, weil

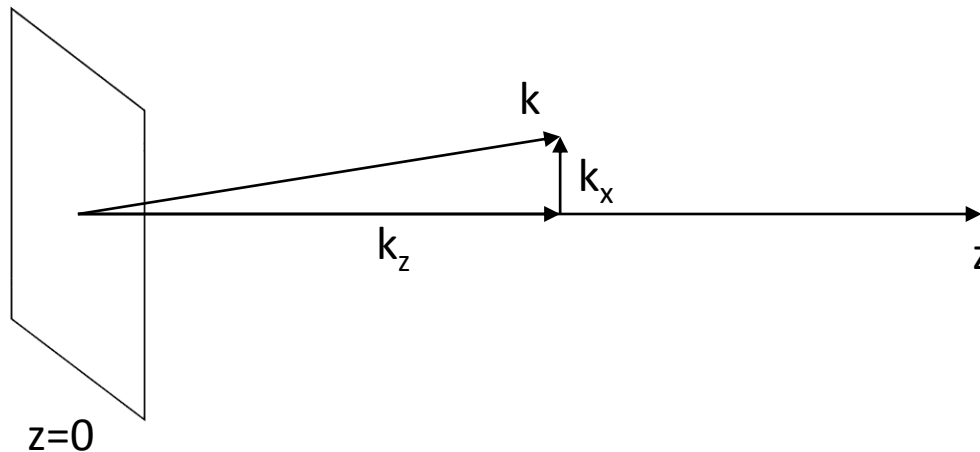
Die paraxiale Näherung

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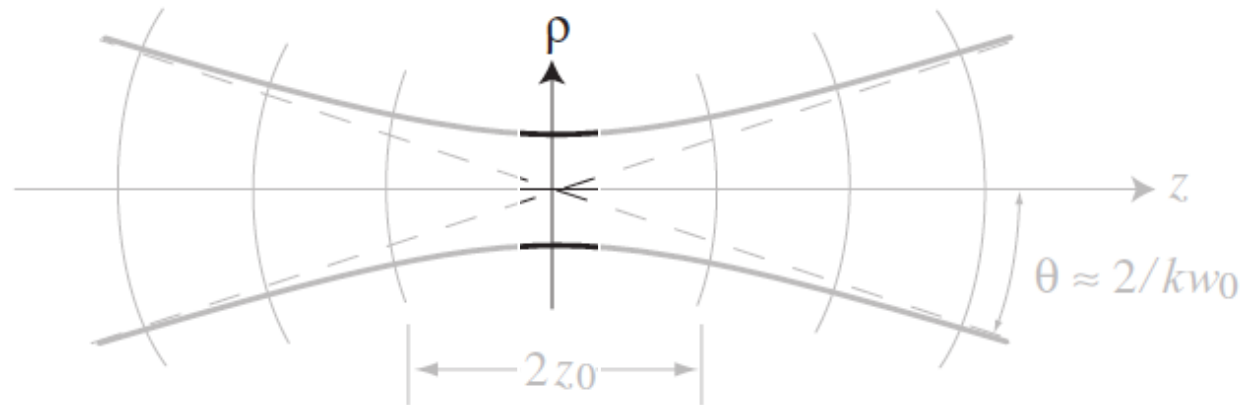
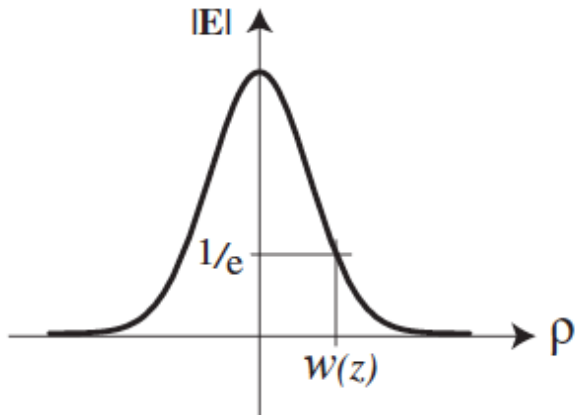
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Annahme: Felder breiten sich vorwiegend in z-Richtung aus ($k_z \gg k_x, k_y$)

$$k_z = k \sqrt{1 - (k_x^2 + k_y^2)/k^2} \approx k -$$

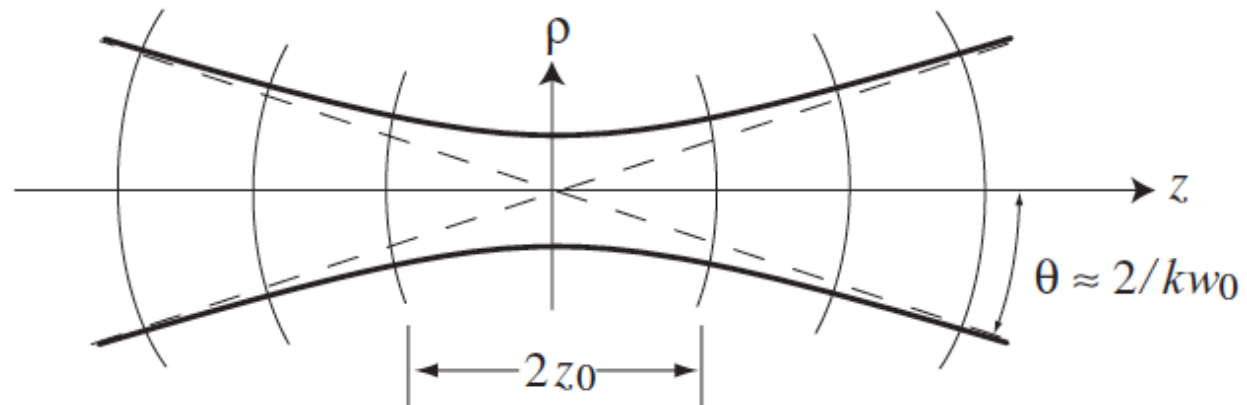
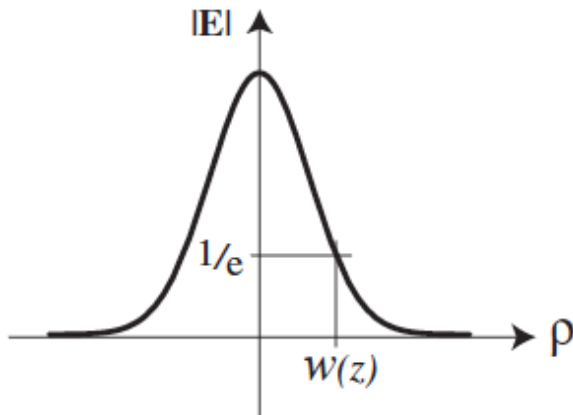


Der Gauss'sche Strahl



Feld in der Fokusebene $z=0$:
$$\mathbf{E}(x', y', 0) = \mathbf{E}_o e^{-(x'^2 + y'^2)/w_0^2}$$

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$$\mathbf{E}(\rho, z) = \mathbf{E}_o \frac{w_o}{w(z)} e^{-\frac{\rho^2}{w^2(z)}} e^{i[kz - \eta(z) + k\rho^2/2R(z)]}$$

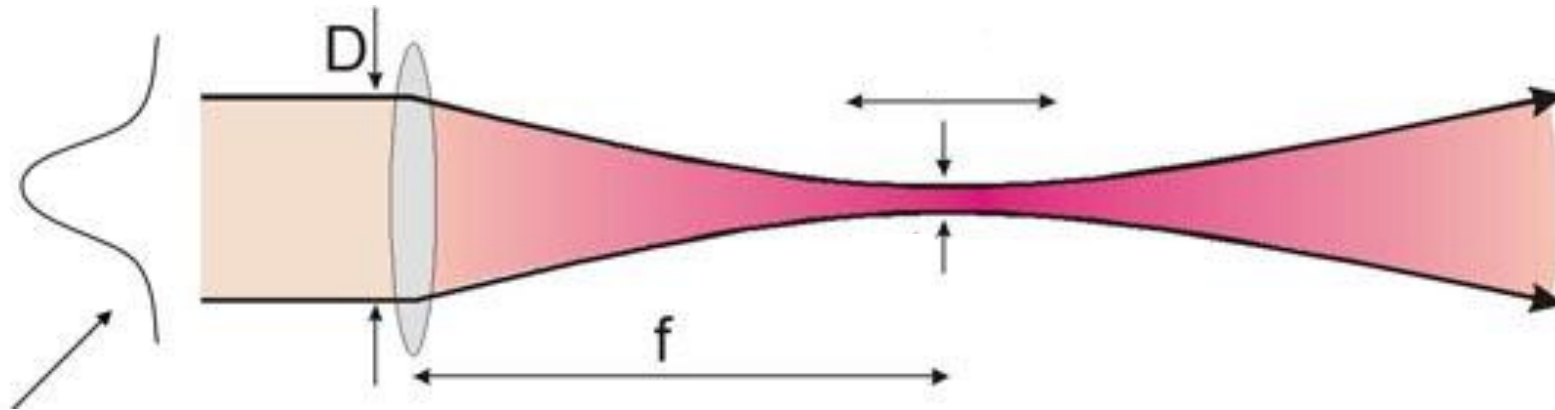
$$w(z) = w_o (1 + z^2/z_o^2)^{1/2} \quad \text{Strahlradius}$$

$$R(z) = z (1 + z_o^2/z^2) \quad \text{Wellenfront Radius}$$

$$\eta(z) = \arctan z/z_o \quad \text{Phasen Korrektur (Gouy Phasen Versatz)}$$

$$z_o = \frac{k w_o^2}{2} \quad \text{Rayleigh Länge}$$

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Der Gauss'sche Strahl



$$z_o = \frac{k w_o^2}{2} \quad \theta = \frac{2}{k w_o}$$

Der Gauss'sche Strahl hat einen freien Parameter!
Welchen?

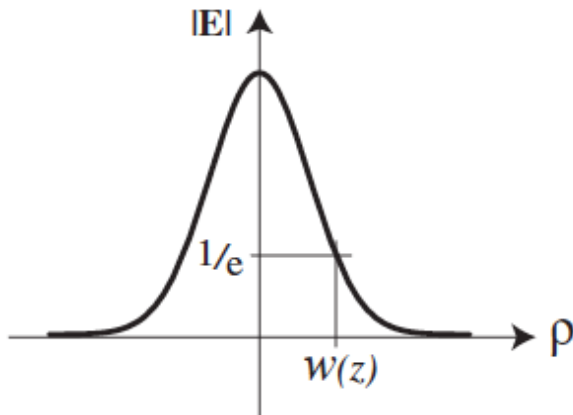
Der Gauss'sche Strahl



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- Erfüllt der Gauss'sche Strahl die Maxwell-Gleichungen?
[redacted]
- Enthält der Gauss'sche Strahl evaneszente Komponenten?
[redacted]

Der Gauss'sche Strahl



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- Erfüllt der Gauss'sche Strahl die Maxwell-Gleichungen?
Nicht divergenzfrei!
- Enthält der Gauss'sche Strahl evaneszente Komponenten?



Gauss'scher Strahl ist eine Näherung!

(paraxial. Und oft eine gute!)

Fernfeldnäherung

Wenn wir das Feldwinkelspektrum in der Ebene $z=0$ kennen, wie lautet dann das Feld (im Realraum) sehr weit weg?

$$\mathbf{E}(x, y, z) = \iint_{-\infty}^{\infty} \hat{\mathbf{E}}(k_x, k_y; 0) e^{i[k_x x + k_y y \pm k_z z]} dk_x dk_y \quad \longrightarrow \quad \mathbf{r} = \mathbf{r}_\infty ?$$

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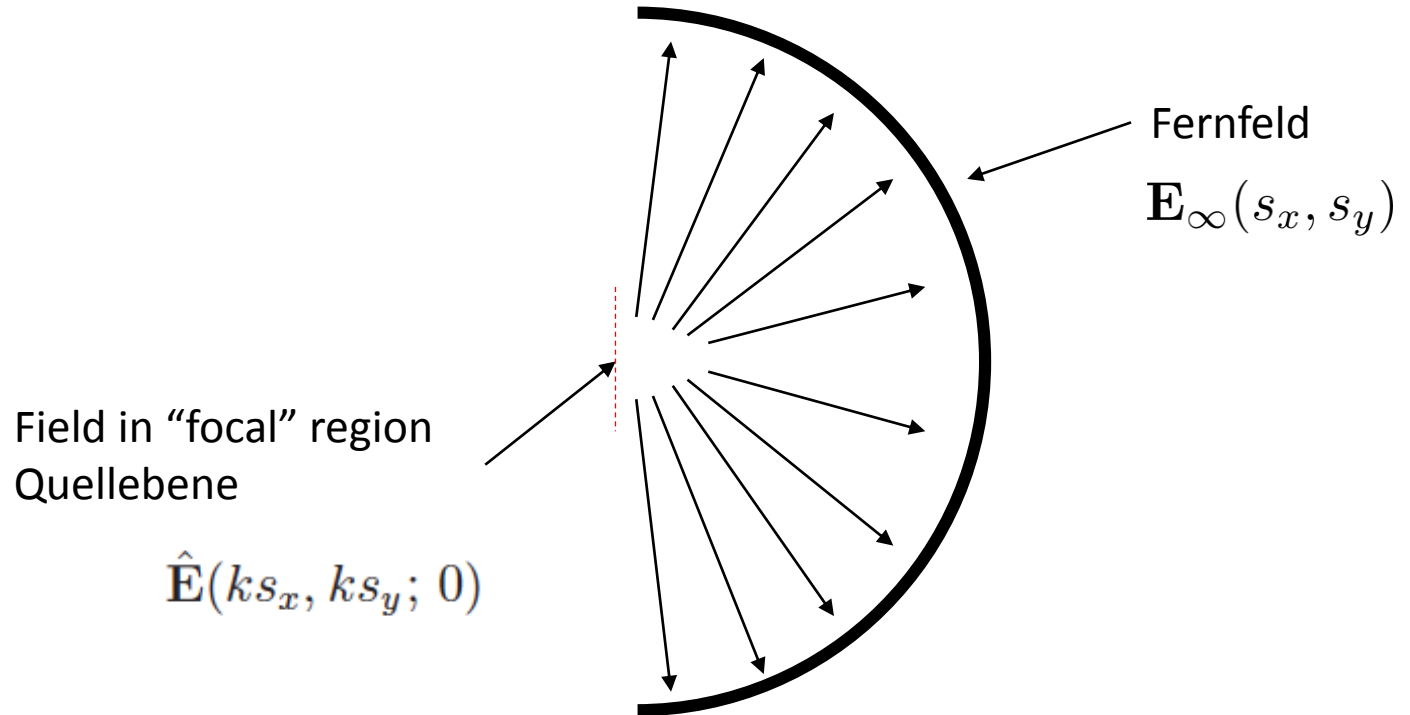


Methode der stationären Phase liefert:

$$\mathbf{E}_\infty(s_x, s_y) = -2\pi i k s_z \hat{\mathbf{E}}(k s_x, k s_y; 0) \frac{e^{i k r}}{r}$$

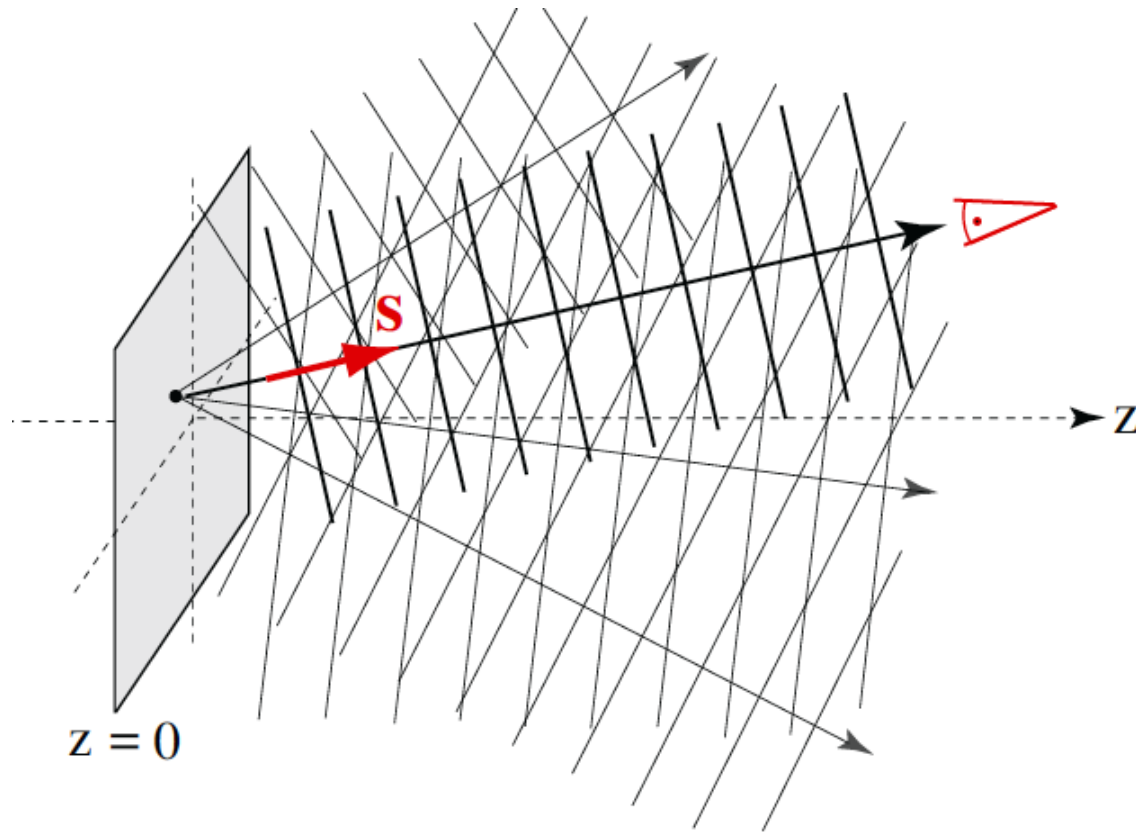
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Fernfeldnäherung führt zur Strahlenoptik



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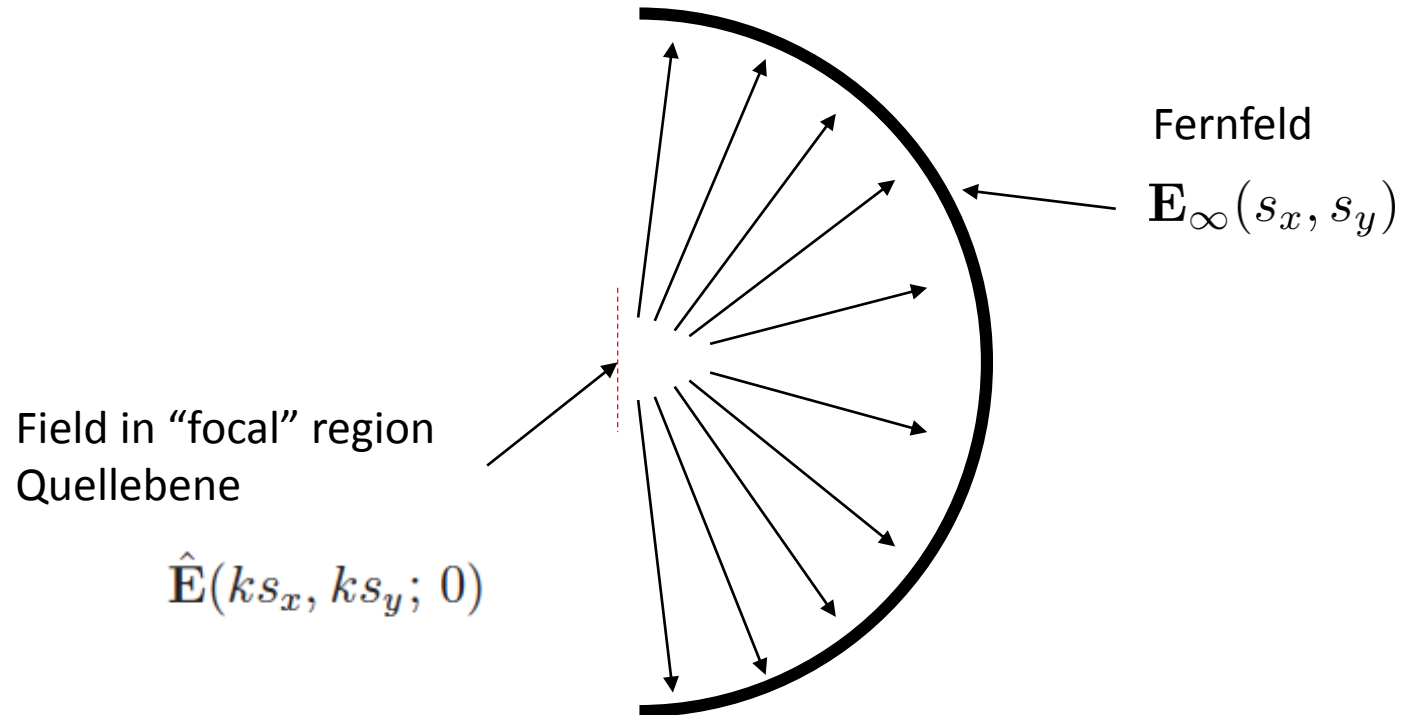
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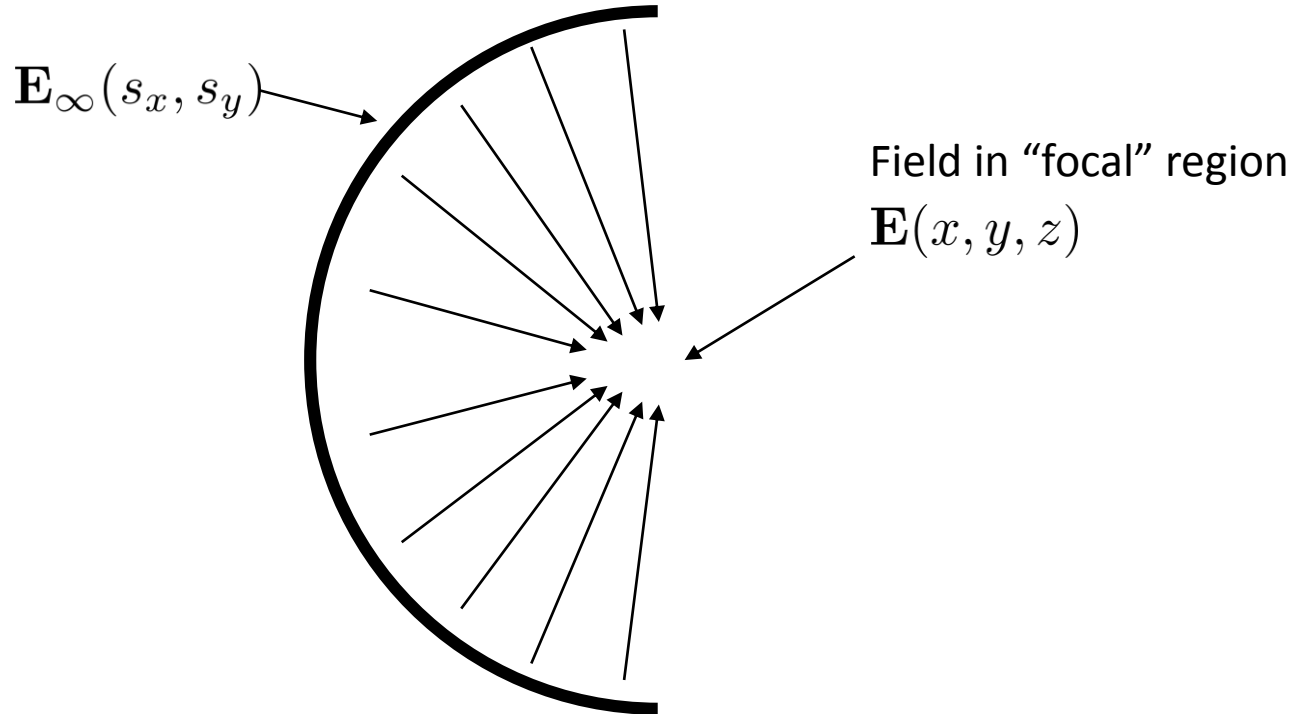
Fourier-Optik

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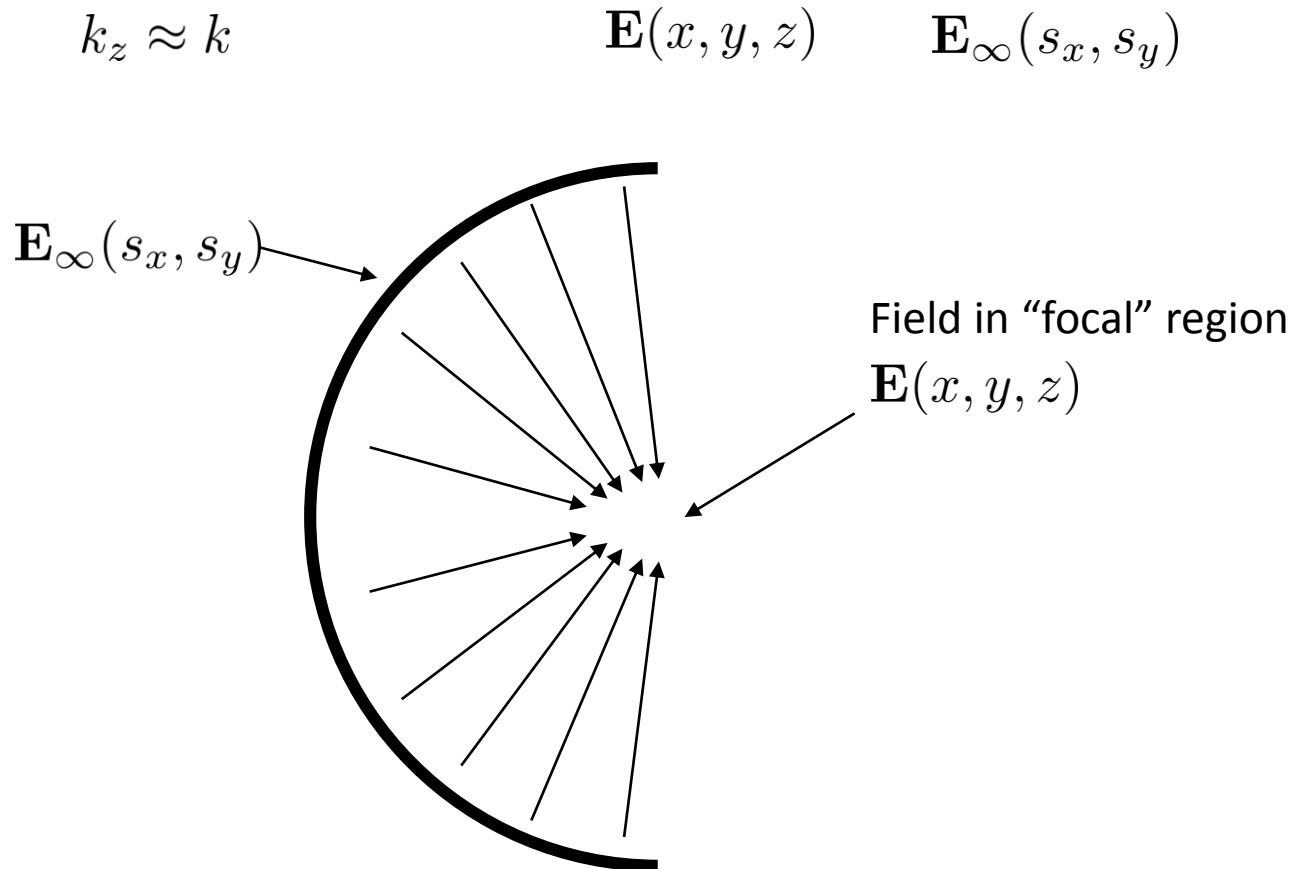


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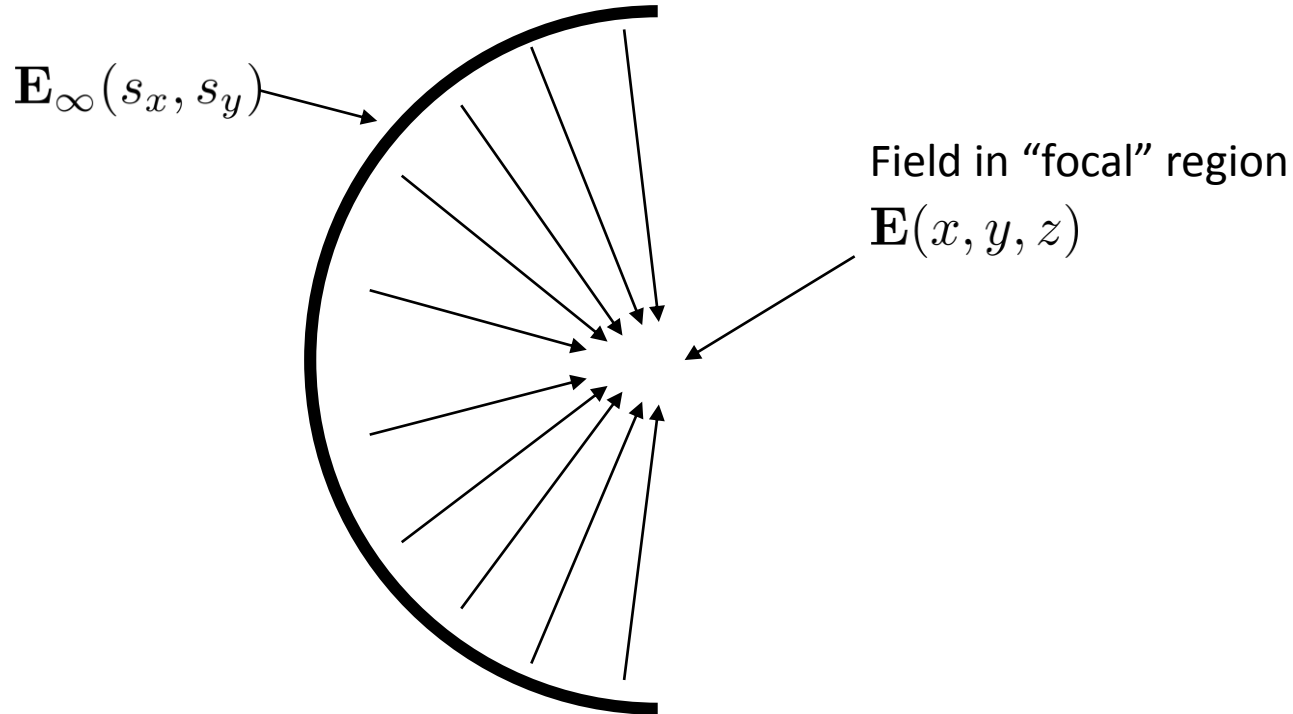
Fourier-Optik



$$\mathbf{E}(x, y, z) = \frac{ir e^{-ikr}}{2\pi} \iint_{(k_x^2 + k_y^2) \leq k^2} \mathbf{E}_\infty\left(\frac{k_x}{k}, \frac{k_y}{k}\right) e^{i[k_x x + k_y y \pm k_z z]} \frac{1}{k_z} dk_x dk_y$$

Fourier-Optik

Im Limes $k_z \approx k$ bilden die Felder $\mathbf{E}(x, y, z)$ und $\mathbf{E}_\infty(s_x, s_y)$ ein Fouriertransformationspaar.



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