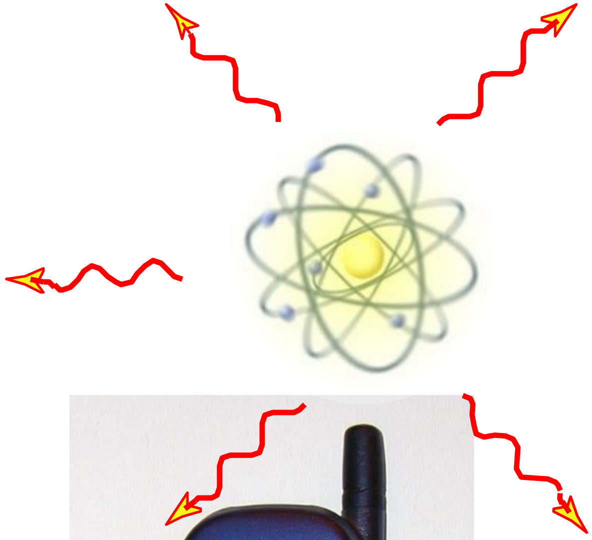
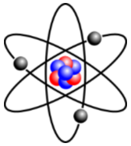
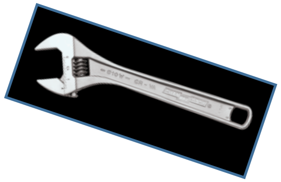
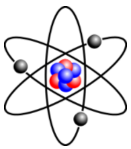
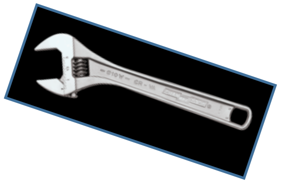
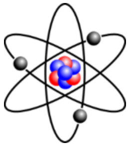
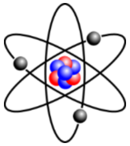
Wie entsteht Strahlung?



Fahrplan

- Die inhomogene Wellengleichung 
- Die Green'sche Funktion 
- Der elektrische Dipol – der Vater aller Strahlungsquellen 
- Skalar- und Vektorpotential 
  - Die Eichungen
- Die Green'sche Funktion des elektrischen Dipols 
- Das Strahlungsfeld des elektrischen Dipols 

Wellengleichung mit Quellterm

Inhomogene Wellengleichung:

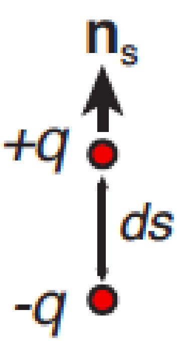
$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial}{\partial t}\left(\mathbf{j} + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M}\right)$$

Im monochromatischen Fall:

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - k^2 \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\mu(\omega)\boldsymbol{j}_0(\boldsymbol{r})$$

Für welche Inhomogenität (Quellterm) sollen wir diese (und jede andere lineare Differential-) Gleichung lösen?

Für die δ-Inhomogenität!



$$\mathbf{j}(\mathbf{r},t) = \frac{\partial}{\partial t}\mathbf{p}(t)\,\delta(\mathbf{r}-\mathbf{r}_0) \longrightarrow \mathbf{j}(\mathbf{r}) = -i\omega\,\mathbf{p}\,\delta(\mathbf{r}-\mathbf{r}_0)$$

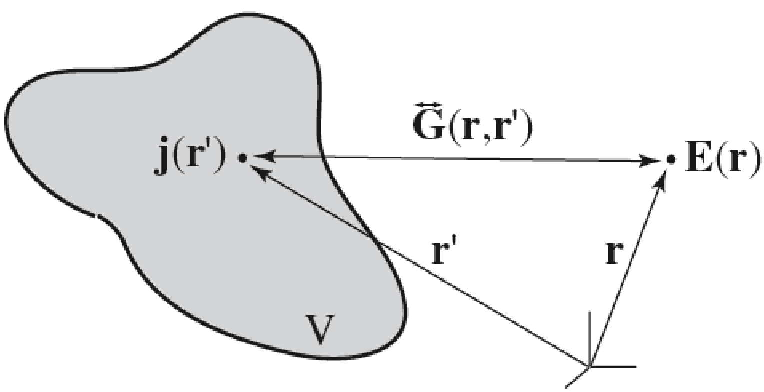
Green'sche Funktion der Wellengleichung

$$\nabla \times \nabla \times \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}) - k^2 \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\mu(\omega)\underbrace{(-\mathrm{i}\omega p)\,\mathbb{1}\,\delta(\boldsymbol{r}-\boldsymbol{r}')}_{\mathbf{j}(\mathbf{r}) \;=\; -i\omega\,\mathbf{p}\,\delta(\mathbf{r}-\mathbf{r}_0)}$$

Mit **G** können wir die Feldverteilung E jeder beliebigen Stromverteilung berechnen!

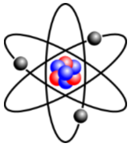
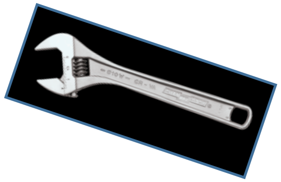
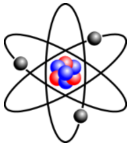
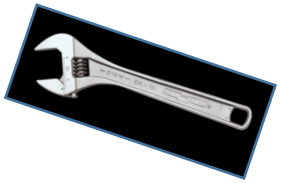
Wie lautet G?

Superpositionsprinzip gilt, da Wellenoperator linear!



$$\longrightarrow \quad \boxed{\mathbf{E}(\mathbf{r}) \;=\; \omega^2\mu_0\mu\,\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_0)\,\mathbf{p}}$$

Fahrplan

- Die inhomogene Wellengleichung 
- Die Green'sche Funktion 
- Der elektrische Dipol – der Vater aller Strahlungsquellen 
- Skalar- und Vektorpotential 
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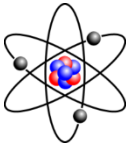
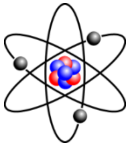
- Die Green'sche Funktion des elektrischen Dipols 
- Das Strahlungsfeld des elektrischen Dipols 

Potentiale im monochromatischen Fall

Im monochromatischen Fall und in der Lorenzeichung: $\nabla \cdot \boldsymbol{A} = \mathrm{i}\frac{k^2}{\omega}\phi$

$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\boldsymbol{A}(\boldsymbol{r}) - \nabla\phi$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\left[\nabla^2 + k^2\right]\mathbf{A}(\mathbf{r}) = -\mu_0\mu\,\mathbf{j}_0(\mathbf{r})$$

$$\left[\nabla^2 + k^2\right]\phi(\mathbf{r}) = -\frac{1}{\varepsilon_0\varepsilon}\rho_0(\mathbf{r})$$

$$\left[\nabla^2 + k^2\right]A_l(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\mu\delta(\boldsymbol{r} - \boldsymbol{r}')\,p_l$$

$$\left[\nabla^2 + k^2\right]G_0(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}')$$

Skalare Gleichung!
Helmholtz-Operator!

Überblick

Inhomogene Wellengleichung des reellen E-Feldes:

$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial}{\partial t}\left(\mathbf{j} + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M}\right)$$

Zeitharmonischer Fall:

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - k^2 \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\mu(\omega)\boldsymbol{j}_0(\boldsymbol{r})$$

Potentiale und Lorenz-Eichung

$$\left[\nabla^2 + k^2\right]\mathbf{A}(\mathbf{r}) = -\mu_0\mu\,\mathbf{j}_0(\mathbf{r})$$

Dipol

$$\mathbf{j}(\mathbf{r}) = -i\omega\,\mathbf{p}\,\delta(\mathbf{r}-\mathbf{r}_0)$$

Green'sches Kalkül

$$\left[\nabla^2 + k^2\right]G_0(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}')$$

Skalare Green'sche Funktion

$$G_0(\mathbf{r},\mathbf{r}') = \frac{\mathrm{e}^{i\,k\,|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

Überblick

Inhomogene Wellen

$$\nabla \times \nabla \times \mathbf{E} + \ldots \nabla \times \mathbf{M}\Big)$$

Zei

$$\nabla \times \nabla \times \ldots (\omega) \boldsymbol{j}_0(\boldsymbol{r})$$

Pot

$$[\nabla^2 + k^2]\, A \ldots -i\omega\, \mathbf{p}\, \delta(\mathbf{r}-\mathbf{r}_0)$$

Gr

$$[\nabla^2 + k^2]\, G \ldots (\mathbf{r},\mathbf{r}') = \frac{e^{i\,k\,|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

Überblick

Inhomogene Wellengleichun

$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2}\frac{\partial}{\partial} \quad \ldots \quad + \nabla \times \mathbf{M}\Big)$$

Zeitharmo

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r} \quad \ldots \quad \mu(\omega)\boldsymbol{j}_0(\boldsymbol{r})$$

Potentiale

$$[\nabla^2 + k^2]\,\mathbf{A}(\mathbf{r}) = \quad \ldots \quad = -i\omega\,\mathbf{p}\,\delta(\mathbf{r}-\mathbf{r}_0)$$

Green'sch

$$[\nabla^2 + k^2]\,G_0(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}')$$

$$G_0(\mathbf{r},\mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

Und jetzt zurück zu den Feldern

Skalare Green Funktion:

$$G_0(\mathbf{r},\mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

Vektorpotential:

$$\boldsymbol{A}(\boldsymbol{r}) = -\mathrm{i}\omega\mu_0\mu G_0(\boldsymbol{r}-\boldsymbol{r}')\,\boldsymbol{p}$$

Per Definition:

$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\boldsymbol{A}(\boldsymbol{r}) - \nabla\phi \qquad \boldsymbol{B} = \nabla\times\boldsymbol{A}$$

Lorenz Eichung:

$$\nabla\cdot\boldsymbol{A} = \mathrm{i}\frac{k^2}{\omega}\phi$$

Magnetisches Feld:

$$\mathbf{H}(\mathbf{r}) = -i\omega\left[\nabla\times\overleftrightarrow{\mathbf{G}}_0(\mathbf{r},\mathbf{r}')\right]\mathbf{p}$$

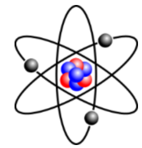
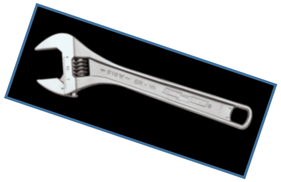
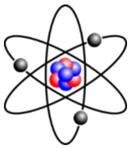
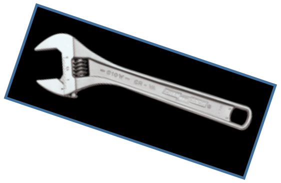
Elektrisches Feld:

$$\mathbf{E}(\mathbf{r}) = \omega^2\mu_0\mu\,\overleftrightarrow{\mathbf{G}}_0(\mathbf{r},\mathbf{r}')\,\mathbf{p}$$

Mit der dyadischen Green-Fkt.:

$$\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left[\overleftrightarrow{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\right]G_o(\mathbf{r},\mathbf{r}')$$

Fahrplan

- Die inhomogene Wellengleichung 
- Die Green'sche Funktion 
- Der elektrische Dipol – der Vater aller Strahlungsquellen 
- Skalar- und Vektorpotential 
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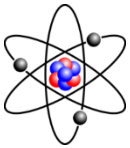
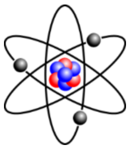
- Die Green'sche Funktion des elektrischen Dipols 
- Das Strahlungsfeld des elektrischen Dipols 

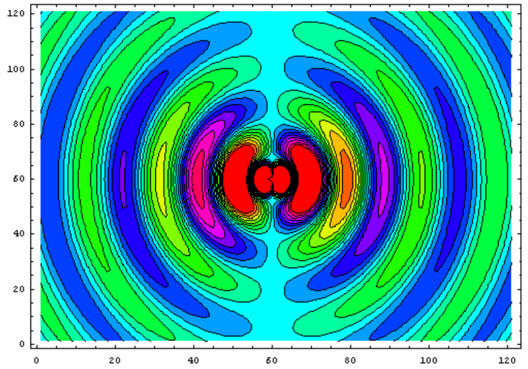
Green'sche Funktion in kartesischen Koordinaten

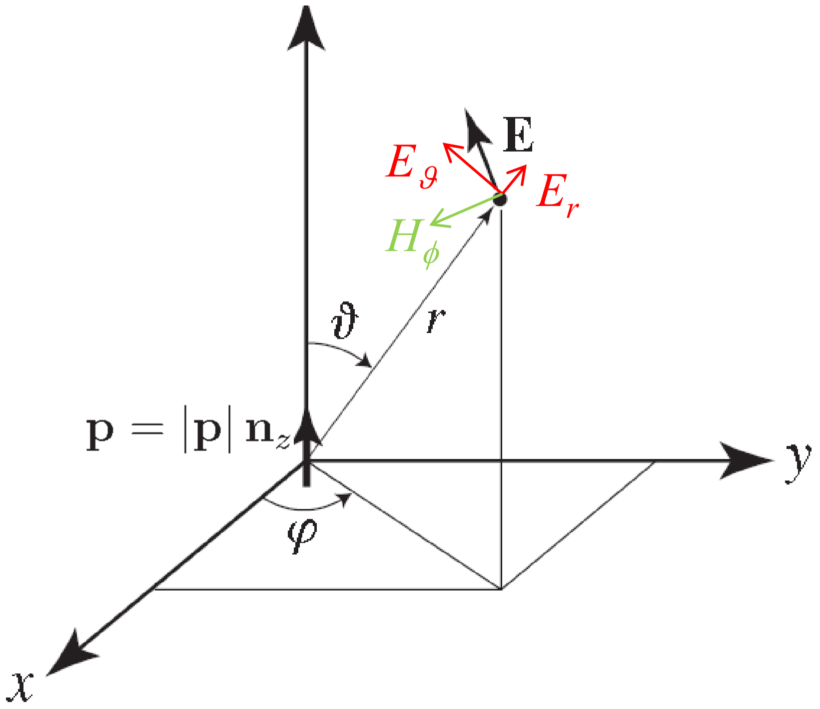
Mit der dyadischen Green-Fkt.: $\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left[\overleftrightarrow{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\right] G_o(\mathbf{r},\mathbf{r}')$

In kartesischen Koordinaten:

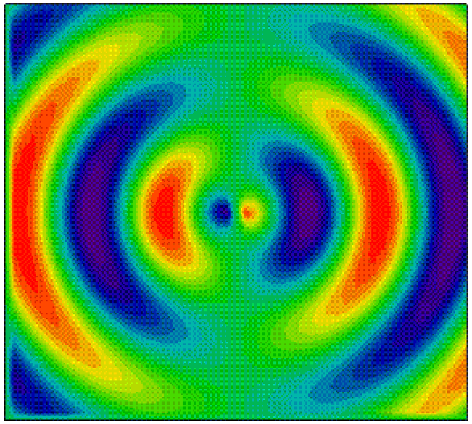
$$\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_o) = \frac{\exp(ikR)}{4\pi R}\left[\left(1 + \frac{ikR-1}{k^2R^2}\right)\overleftrightarrow{\mathbf{I}} + \frac{3-3ikR-k^2R^2}{k^2R^2}\frac{\mathbf{RR}}{R^2}\right]$$

Dipol-Strahlung für z-orientierten Dipol





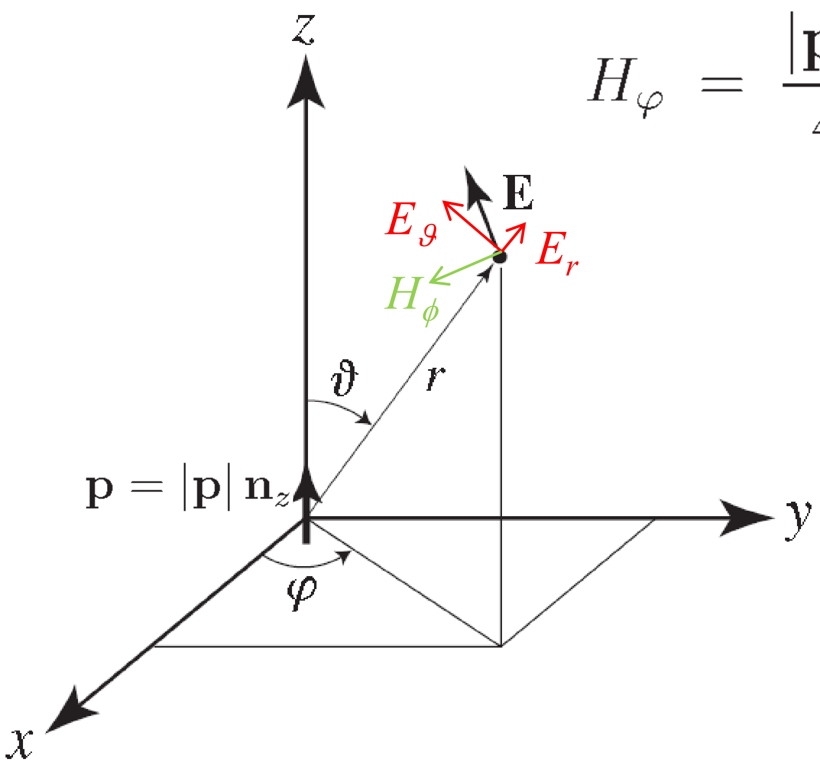
Dipol-Strahlung für z-orientierten Dipol



$$E_r = \frac{|\mathbf{p}|\cos\vartheta}{4\pi\varepsilon_0\varepsilon}\frac{\exp(ikr)}{r}k^2\left[\underbrace{\frac{2}{k^2r^2}}_{\text{NF}} - \underbrace{\frac{2i}{kr}}_{\text{IF}}\right],$$

$$E_\vartheta = \frac{|\mathbf{p}|\sin\vartheta}{4\pi\varepsilon_0\varepsilon}\frac{\exp(ikr)}{r}k^2\left[\underbrace{\frac{1}{k^2r^2}}_{\text{NF}} - \underbrace{\frac{i}{kr}}_{\text{IF}} - \underbrace{1}_{\text{FF}}\right],$$

$$H_\varphi = \frac{|\mathbf{p}|\sin\vartheta}{4\pi\varepsilon_0\varepsilon}\frac{\exp(ikr)}{r}k^2\left[-\underbrace{\frac{i}{kr}}_{\text{IF}} - \underbrace{1}_{\text{FF}}\right]\sqrt{\frac{\varepsilon_0\varepsilon}{\mu_0\mu}}$$



NB:

- Magnetfeld hat keinen Nahfeldterm
- Fernfelder sind transversal
- Intermediate Field ist 90° ausser Phase mit Nah- und Fernfeld